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Production Function Estimation: Biased Coefficients and Endogenous Regressors, or a Case of Collective Amnesia?

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ABSTRACT

The possible endogeneity of labor and capital in production functions, and the consequent bias of the estimated elasticities, has been discussed and addressed in the literature in different ways since the 1940s. This paper revisits an argument first outlined in the 1950s, which questioned production function estimations. This argument is that output, capital, and employment are linked through a distribution accounting identity, a key point that the recent literature has overlooked. This identity can be rewritten as a form that resembles a production function (Cobb-Douglas, CES, translog). We show that this happens because the data used in empirical exercises are value (monetary) data, not physical quantities. The argument has clear predictions about the size of the factor elasticities and about what is commonly interpreted as the bias of the estimated elasticities. To test these predictions, we estimate a typical Cobb-Douglas function using five estimators and show that: (i) the identity is responsible for the fact that the elasticities must be the factor shares; (ii) the bias of the estimated elasticities (i.e., departure from the factor shares) is, in reality, caused by the omission of a term in the identity. However, unlike in the standard omitted-variable bias problem, here the omitted term is known; and (iii) the estimation method is a second-order issue. Estimation methods that theoretically deal with endogeneity, including the most recent ones, cannot solve this problem. We conclude that the use of monetary values rather than physical data poses an insoluble problem for the estimation of production functions. This is, consequently, far more serious than any supposed endogeneity problems.

KEYWORDS: Accounting Identity; Endogeneity; Monetary Values; Production Functions; Total Factor Productivity

JEL CLASSIFICATIONS: C18; C81; C82

1. INTRODUCTION

Ever since Marschak and Andrews' (1944) seminal paper, the literature on production function estimation has tried to deal with the possible bias of the estimated coefficients, caused by the likely endogeneity of the regressors. The simplest case is a Cobb-Douglas value-added production function: [1]

$$Q_{it} = A_{it} L_{it}^{\alpha} K_{it}^{\beta} \quad (1)$$

where Q, L, and K denote physical output, employment, and the physical capital stock, respectively; $A_{it} = e^{\pi_0 + \varepsilon_{it}}$ is some measure of technology, in general thought to be directly unobservable to the researcher, the subscript i denotes firm/sector, and subscript t refers to time. π_0 is the mean efficiency level across firms and over time and ε_{it} is time and firm-/sector-specific deviation from the mean.

The ordinary least squares (OLS) estimator of equation (1),

$$ln\, Q_{it} = \pi_0 + \alpha\, ln\, L_{it} + \beta\, ln\, K_{it} + \varepsilon_{it} \quad (2)$$

where $(\pi_0 + \varepsilon_{it})$ is the firm-level productivity, will produce biased estimates of α and β if regressors and the error term are correlated, i.e., $E[L_{it}\varepsilon_{it}] \neq 0$ or $E[K_{it}\varepsilon_{it}] \neq 0$. The bias of the OLS estimates results from the potential contemporaneous correlation between the inputs (i.e., factors of production) and the unobserved firm-specific productivity shocks. This to say, the error distribution cannot be considered to be independent of the regressors' distribution. Consequently, $E[X\,\varepsilon] \neq 0$, where X is the regressor (labor and capital). This may result from the fact that firms that experience an exogenous productivity shock may respond by altering the amount of inputs

[1] We acknowledge that other production functions (e.g., CES, translog) are also estimated in some applications. There is, for example, a literature on the elasticity of substitution that seems to indicate that this is less than one. This issue is not addressed here. As shown in section 3, the exact specification of the production function is not relevant to the main argument of this paper, and the Cobb-Douglas function is still widely used.

they use.[2] This is an important issue, *per se*, as for example, unbiased estimates of the output elasticities are required to accurately estimate total factor productivity (TFP), i.e., $lnTFP_{it} = \hat{\pi}_0 + \hat{\varepsilon}_{it} = ln\, Q_{it} - \hat{\alpha}\, ln\, L_{it} - \hat{\beta}\, ln\, K_{it}$.

Two early methods used to deal with this problem are instrumental variables (IV) and least squares dummy variables (LSDV) (Mundlak 1961).[3] The former uses instruments that must be correlated with the explanatory variables, but uncorrelated with the error term; the latter uses sector-specific, i.e., within-firm variation in the sample, but time-invariant variables (fixed effects estimator). The last twenty-five years have seen the development of new estimation techniques to deal with this problem, specifically in the context of production functions. These have been used with large, firm-level datasets (e.g., Olley and Pakes 1996; Blundell and Bond 2000; Levinsohn and Petrin 2003).[4]

This paper discusses and elaborates upon a potentially important, yet virtually overlooked, problem with the estimation of production functions first mentioned in the literature in the 1950s and 1960s. A series of papers argued back then that all that estimations of production functions do is capture a transformation of a distribution accounting identity. This is that value added is definitionally equal to the total compensation of labor plus total profits. The core of this problem appears, on the surface, to be only indirectly related to that of regressors' endogeneity in production functions. However, the identity argument provides a potentially powerful rationale to properly understand this issue, although not by proposing an alternative estimator. Beyond this, there is an additional issue of utmost importance, which concerns the testability (i.e., the possibility of rejecting) of the production function.

[2] The general finding is that the elasticity of employment will be biased upward and that of capital downward. Van Beveren (2012) provides a comprehensive literature review of other problems that affect the estimation of production functions (as well as of other estimators to deal with endogeneity in production functions). These problems include the bias due to the entry and exit of firms, the use of price deflators to proxy for firm-level prices, and the pervasiveness of multiproduct firms in manufacturing.

[3] It should be noted, however, that Zellner, Kamenta, and Dreze (1966) argued that as the production function contains an error term, output is stochastic and the firm should maximize expected profits, not actual profits. They posit a model in which profits are stochastic and where the objective function is the maximization of the mathematical expectation of profits. They show that the production function in the simultaneous equation model should be estimated by OLS.

[4] See the comprehensive surveys by Ackerberg et al. (2007), Van Beveren (2012), and Rovigatti and Mollisi (2018).

To understand the problem, it should be noted that it has been generally accepted since Cobb and Douglas (1928) that a production function is a technological relationship between *physical* inputs and *physical* output. Discussions of the endogeneity problem, and of other econometric issues, proceed *as if* the input and output data in empirical applications were physical quantities, i.e., the production function estimated is $Q = f(A, L, K)$, where, as above, Q, L, and K denote physical output, employment, and the physical capital stock, respectively (any functional form); A is some measure of technology, in general thought to be directly unobservable to the researcher. This was an assumption of Cobb and Douglas (1928, 139), where they explicitly referred to the *volume of physical production*.

Yet, in empirical applications, some measure of deflated monetary values is used because physical data are not available, as acknowledged, for example, by Levinsohn and Petrin (2003), Van Beveren (2012), or Ackerberg et al. (2015). This applies to output, capital stock, and materials.[5] Because of the use of value (monetary) data, what is most often estimated is not the physical production function but $V = f(A, L, J)$, where V and J are the deflated values (i.e., real) of value added and capital, respectively. For total (or gross) output (Y), the production function is $Y = f(A, L, J, Z)$, where Z denotes materials. Again, total output and materials are also measured as deflated monetary values. This is true (i.e., the use of value data) even in applications with firm-level data. While researchers today appreciate the difference between physical and value data, their objective is to devise estimation strategies to obtain the true elasticities (i.e., those with physical data) through the use of monetary values by imposing strong assumptions. The reality, however, is that the use of monetary values has important ramifications for the estimation of production functions that seem to have escaped their attention.

What is the accounting identity problem? As we will discuss below, the series V, L, J, and also implicitly A, are all linked through the distribution accounting identity $V \equiv W + P$, where W is

[5] Employment is the only physical measure (the number of workers or hours worked, proxying the flow of labor services). Output (except perhaps in the limited case of some agricultural output) and capital are not measured in physical (homogenous machines) units. Often the capital stock is calculated using the perpetual inventory method and constant price value data (e.g., Levinsohn and Petrin 2003, 324). There is another strand of the literature, which dates back to the 1940s, that is very critical of the notion of production function, in particular of the *aggregate* production function. This is the classical literature on the problem of *aggregation* in production functions. See, for example, the survey by Felipe and Fisher (2003). The use of firm-level data recently is directed at overcoming this criticism. This, however, does not avoid the problems discussed in this paper, namely the use of monetary values.

the total wage bill and P denotes total profits. Likewise, there is a corresponding identity for gross (or total) output including intermediate materials. They key point is that these variables are measured in monetary (constant price) values, not in physical quantities. The identity can be rewritten as $V \equiv f(M, L, J)$, with M to be defined later (we will show below why L and J appear in this transformation), and f is a function that resembles a production function (Cobb-Douglas, CES, translog, etc.), but it merely shows the division of the deflated value of output (or income) between employment and capital. This transformation has a series of implications that we test. The most important ones are, first, that the estimated elasticities must be the factor shares and, second, that there is no endogeneity bias that results from the correlation between regressors and error term. Instead, estimates of the employment and capital elasticities are biased because researchers proxy M with A (e.g., time trend, human capital) and estimate $V = f(A, L, J)$.

Phelps-Brown (1957) was possibly the first author to articulate the problem in his discussion of Cobb and Douglas (1928). Hogan (1958) noted this difficulty in a comment on Solow (1957). Subsequent papers by Simon and Levy (1963), Simon (1979a), and Samuelson (1979) also rehearsed the argument in various forms. Simon thought that the argument was important enough to refer to it in its Nobel Prize lecture (Simon 1979b, 497). Samuelson (1979, 934) wrote that he left it to others to empirically evaluate his arguments: "I hope that someone skilled in econometrics and labor will audit and evaluate my critical findings." Yet this unresolved issue seems to have been largely forgotten. This paper considers the accounting identity argument and its main implications, specifically that efforts to solve "endogeneity" concerns continue to run afoul of the identity problem.

The remainder of the paper is structured as follows. To motivate our work, section 2 presents estimates of the elasticities of a Cobb-Douglas production function using five estimators: ordinary least squares (OLS), fixed effects least-square dummy variables (LSDV), instrumental variables (IV), dynamic system generalized method of moments (GMM), and the Levinsohn and Petrin (L-P) procedure. Section 3 presents the accounting identity argument for both cross-sectional and time-series data and considers the estimation bias in the light of the identity. As noted above, this is the result of omitting (or incorrectly approximating) a term from the distribution identity when this is estimated as a production function. This is not the standard endogeneity problem that results from the error term being correlated with labor and capital. We also discuss the difference with physical data. Section 4 discusses the estimation results of the accounting identity rewritten into a form that looks like a Cobb-Douglas function, using the same five estimators. We show that all estimation methods yield essentially the same results. Section 5 concludes by emphasizing that the endogeneity problem, as understood for decades, is a figment. Appendix 1 explains the dataset. Appendix 2 shows the results for the gross output production function. Appendix 3 shows the results with an alternative profit rate. Finally, appendix 4 provides estimates of the coefficients of the identity using Kalman filter estimation and discusses the relationship between the identity and the CES production function.

2. PRODUCTION FUNCTION ESTIMATES: BASELINE RESULTS

To motivate the discussion, table 1 shows typical estimation results of a Cobb-Douglas production function using the value data that are available to researchers (results with total output are shown in appendix 2). The table shows five sets of results, corresponding to five methods: OLS, plus four methods that in principle could deal (in different ways) with the endogeneity problem—LSDV, IV, GMM, and the L-P procedure. While LSDV, IV, and GMM are well known and can be applied in any setting, the L-P method was developed specifically to deal with the endogeneity problem in production functions (Levinsohn and Petrin 2003).[6] We acknowledge that other methods have been developed recently expanding the control function approach.[7] The five estimators we use are

[6] L-P is a semiparametric estimator that solves the simultaneity problem of the regressors by using intermediate inputs to proxy for unobserved productivity shocks. This proxy controls for the part of the error correlated with inputs by eliminating any variation that is related to the productivity term. Capital is assumed to be the state variable and employment the freely variable. In this method, the error term (ε) of the production function is decomposed into a transmitted productivity shock (θ) and an i.i.d. error term that is uncorrelated with input choices, i.e., has no impact on the firm's decisions (u), i.e., $\varepsilon = \theta + u$. θ is a state variable that impacts the firm's decisions rules. It is not directly observed by the researcher and it can affect the choices of inputs. The method works in two steps. The first one is an OLS regression that adds a third-order polynomial in capital (J) and materials (Z): $ln\, V_{it} = \delta_0 + \alpha\, ln\, L_{it} + \left[\sum_{b=0}^{3}\sum_{c=0}^{3-b}\delta_{bc}\, ln\, J_{it}^{b}\, ln\, Z_{it}^{c}\right] + u_{it}$. This first step yields the elasticity of labor ($\hat{\alpha}$). The elasticity of capital, β, is obtained in the second step. This consists in minimizing the sample residual (imposing the elasticity of labor obtained in the first step, $\hat{\alpha}$), obtained from the first step: $\min_{\beta^*} \sum_{it}\left(ln\, V_{it} - \hat{\alpha}\, ln\, L_{it} - \beta^*\, ln\, J_{it} - E\left[\theta_{it}\widehat{|\theta_{it-1}}\right]\right)^2$.

[7] See, for example Ackerberg, Caves, and Frazer (2015), who provide a discussion of the Levinsohn and Petrin (2003) approach for the value-added production function. They highlight the "functional dependence" problem that the method suffers from. This refers to the fact that the moment condition underlying the first stage estimating equation does not identify the labor coefficient. Ackerberg, Caves, and Frazer (2015) show that, under the data generating

easy to implement and that is why we chose them. Moreover, a key point of this paper is that the estimation method is not the issue, and we ask the reader to hold this point until we get to sections 3 and 4. Details about the dataset are provided in appendix 1.

Considering the results produced by the five estimators in table 1, it can be seen that there are significant discrepancies. The null hypothesis of no endogeneity, i.e., that the OLS error term is not correlated with the regressors, was rejected.[8] While estimates of the elasticity of employment produced by the OLS, LSDV, and IV estimators are very similar (0.46–0.47), the value produced by GMM is significantly higher (0.73), and that produced by L-P is much lower (0.37). The estimates of the elasticity of capital with LSDV, IV, and L-P are large (0.70, 0.79).[9] The estimate of capital given by GMM is 0.33, and 0.54 by OLS. Returns to scale with LSDV and IV are very high, while OLS indicates constant returns. Finally, the OLS estimate of employment elasticity (0.47) is in line with those given by LSDV and IV. However, the estimate of the capital elasticity (0.54) is much lower than those given by the other estimators, except GMM (much lower). The OLS degree of returns to scale is the closest to constant.

These results (elasticities) are qualitatively similar to those obtained by Van Biesebroeck (2008: table 2) also using different estimators, i.e., some implausible elasticities (and consequently returns to scale), which led him to add that "there is no agreement whatsoever as to the true labor coefficient" (p. 317).

Under a conventional (i.e., endogeneity) interpretation, the conclusion up to here would be that it is difficult to ascertain which elasticities are the correct ones, and one would have to agree with Van Beveren (2012, 99; italics in original) that "…in practice evaluation of factor elasticities is

processes that are consistent with the stated assumptions of the L-P model, labor is a deterministic function of the set of variables that, in the L-P procedure, need to be nonparametrically conditioned on. Once one does this nonparametric conditioning, there is no variation in labor left to identify the labor coefficient. Gandhi, Navarro, and Rivers (2020) extend the Ackerberg, Caves, and Frazer (2015) analysis to the gross output production function. They argue that the latter also lacks a proper identification restriction. Gandhi, Navarro, and Rivers (2020) identification restriction is a first-order condition assuming perfect competition.

[8] The Durbin-Wu-Hausman test (with inputs lagged one period as instruments) was used to determine if employment and capital are endogenous in the OLS regression.

[9] Ackerberg et al. (2007) note that often LSDV leads to very low estimates of the elasticity of capital. This is not so in our case.

more problematic due to the absence of a definite prior on how high (or low) factor elasticities *should* be." This statement, however, is only partially true. For decades there has been agreement in the profession that "when someone claims that production functions work, he means (a) that they give a good fit to input-output data without the intervention of factor shares and (b) that the function so fitted has *partial derivatives that closely mimic the observed factor shares*" (Solow 1974, 121; italics added).[10] In other words, if the production function "works," the elasticities should be equal to the factor shares. Moreover, both (a) and (b) provide a test of the production function.

[10] The reality is that the profession's prior about the value of the elasticities is the factor shares. See also Douglas (1976, 914) when he claimed that a "considerable body of independent work tends to corroborate the original Cobb-Douglas formula, but, more important, the approximate coincidence of the estimated coefficients with the actual shares received also strengthens the competitive theory of distribution and disproves the Marxian." Elasticities very different from the factor shares would be questioned, and likely thought of as biased estimates.

Table 1. Cobb-Douglas Regressions: Dependent Variable Is the Logarithm of Value Added (lnV)

	OLS	LSDV	IV	System GMM	L-P
	(1)	(2)	(3)	(4)	(5)
Constant	5.00 (94.7)	0.18 (1.77)	0.29 (2.76)	0.70 (6.67)	
Value added lagged (V_{t-1})				0.90 (57.2)	
Employment (L)	0.47 (119)	0.46 (93.3)	0.46 (93.8)	0.73 (7.68)	0.37 (17.8)
Capital (J)	0.54 (165)	0.79 (185)	0.79 (180)	0.33 (4.44)	0.70 (4.90)
Sector fixed effects		Yes	Yes	Yes	
R^2	0.819	0.920	0.921	N.A.	N.A.
Degree of returns to scale	1.017 (325)	1.250 (223)	1.245 (220)	1.057 (13.9)	1.067 (0.25)
No. of observations	25,305	25,305	24,826	24,826	25,305

Source: Authors

Notes: (i) Regressors are expressed in logarithm; (ii) columns (1)–(2): t-values are in parentheses. columns (3)–(5): z-values are in parentheses; (iii) columns (1)–(4): z-values are in parentheses under degrees of returns to scale; (iv) column (4) shows the long-run elasticities of employment and capital (z-values in parenthesis); (v) column (5): chi-squared (Wald test) is in parentheses to test the null that the degree of returns to scale is 1; (vi) LSDV (column 2): $ln\,V_{it} = \pi_0 + \alpha ln\,L_{it} + \beta ln\,J_{it} + \sigma_i + \varepsilon_{it}$, where σ_i denotes sector fixed effects. One-period lagged regressors were also used and obtained very similar results; (vii) IV (column 3): Estimation is two-stage least squares: (a) $ln\,J_{it} = \kappa_0 + \varrho\, ln\,J_{it-1} + \tau\, ln\,L_{it} + \sigma_i + \upsilon_{it}$, where $ln\,J_{it-1}$ is the instrument and $ln\,L_{it}$ is treated as exogenous variable; (b) $ln\,V_{it} = \pi_0 + \alpha\, ln\,L_{it} + \beta\, ln\,\hat{J}_{it} + \sigma_i + \varepsilon_{it}$. A variety of potential instruments were tested but in, most cases, results were poor, e.g., very high, negative, or insignificant elasticities in the second step. Results reported use capital lagged one period as the instrumental variable in the first stage; (viii) GMM (column 4): Estimation is carried out using the one-step system GMM estimator, which contains the equation in levels $ln\,V_{it} = \pi_0 + \rho ln\,V_{it-1} + \alpha ln\,L_{it} + \beta ln\,J_{it} + \sigma_i + \varepsilon_{it}$ and the difference equation with the variables in first difference ($(ln\,V_{it} - ln\,V_{it-1}) = \rho(ln\,V_{it-1} - ln\,V_{it-2}) + \alpha(ln\,L_{it} - ln\,L_{it-1}) + \beta(ln\,J_{it} - ln\,J_{it-1}) + (\varepsilon_{it} - \varepsilon_{it-1})$). Moment conditions: (a) second and further lags of the dependent variable are used as instrument for the residual of the equation in differences; (b) lagged first differences of the dependent variable are used to construct the orthogonality condition for the error term of the equation in levels. Additional orthogonality conditions arise from using lags of the regressors as instruments for the residuals: (c) lagged values of the level of regressors are used as instruments for the regressors in the equation in first differences; (d) lagged values of the difference of regressors are used as instruments to estimate the equation in levels. Results using the two-step GMM estimator (forward orthogonal deviations), i.e., the difference equation is constructed by subtracting from the current observation of each variable, the average of all future observations of that particular variable, are very similar. We used lags 2 through 4 of the levels as instruments for the transformed data and lag 1 of the differences for the levels data, i.e., option *laglimits (2 4)*. Tests for autocorrelation and validity of the instruments: the z-values of AR(1) = -12.43 and AR(2) = -2.37. This means that we reject the null hypothesis that there is no second-order serial correlation. The chi-squared Sargan test = 4,545 and the chi-squared Hansen test = 461. This means that we reject the null hypothesis that the instruments are exogenous. See Baum et al. (2003) and Roodman (2009). When the system is estimated without the lagged dependent variable ($ln\,V_{it-1}$), the elasticity of employment is 0.26, and that of capital 0.78.

While it is true that, *theoretically,* OLS could yield biased estimates of the elasticities, the ones we have obtained do not appear to be *worse* than those produced by the other estimation methods. Estimators to deal with the problem of endogeneity, like GMM or L-P, often yield dubious (e.g., insignificant or very large) elasticities. Finally, all methods tend to produce relatively high statistical fits and, when results seem to be sensible, elasticities add up to about one. The choice of estimator poses a conundrum because there is no objective rationale to decide which estimates are the correct ones.

3. THE DISTRIBUTION ACCOUNTING IDENTITY: STATEMENT OF THE PROBLEM

We reemphasize that researchers do not estimate $Q = f(A, L, K)$ (as noted above, A is some measure of technology). Instead, they use constant-price (monetary) value data and estimate $V = f(A, L, J)$. This section shows the problem that the use of monetary values poses, although it was not a point explicitly stressed by Phelps-Brown (1957), Simon and Levy (1963), Simon (1979a), or Samuelson (1979) in their critique of production functions as approximations to an accounting identity (i.e., they just mentioned that the series are linked through the identity).[11] Likewise, as mentioned in the introduction, authors working today in this field acknowledge explicitly the difference between physical and monetary data for output and the capital stock (e.g., Ackerberg, Caves, and Frazer 2015), yet they have overlooked the fact that the monetary series are definitionally related through an identity. Their identification strategy requires strong assumptions, which might solve an endogeneity problem that would only exist if they were working with physical units (e.g., Gandhi, Navarro, and Rivers 2020).

The accounting identity argument starts by recognizing that the series V, L, and J (and implicitly A) in constant prices or monetary terms is definitionally related through the accounting identity:

[11] The other Nobel Prize winner aware of the identity argument is Robert Solow. Carter (2011) narrates the correspondence between Herbert Simon and Solow in 1971, where the former pointed out to the latter the damaging implications of the identity argument. The letters indicate that Solow was not convinced and eventually ignored the problem.

$$V \equiv W + P \qquad (3)$$

where real value added (V) is the difference between gross output (Y) and intermediate materials (Z) (see below), appropriately deflated. See Jorgenson and Griliches (1967) for a discussion of the consistency of the identity in real terms at the aggregate level.

We now split the real wage bill (W) and real total profits (P) and write the identity as:

$$V \equiv wL + rJ \qquad (4)$$

where w is the average real wage rate and r is the average profit rate. Neither one is assumed to be the marginal product of the corresponding factor of production. Identities (3) and (4) must be satisfied by all datasets relating value added, the wage bill (and wage rate and employment), and profits (and the profit rate and capital), and hold at any level of aggregation, from firm-level to economy-wide level. Output is a value measure (e.g., dollars) and so are the wage bill ($W = wL$) and *all* profits ($P = rJ$). The real wage rate (w) is dollars per worker and employment (L) is the number of workers or hours worked. Their product is, therefore, a value measure (i.e., dollars). The profit rate (r) is a pure number and the capital stock (J) is a dollar measure (not the number of homogeneous machines). Their product is, therefore, a value measure (i.e., dollars). There are no economic or other assumptions about factor markets or the degree of returns to scale. The identity is compatible with any type of market and degree of returns to scale and holds even in case there is no well-defined production function.[12] Appendix 3 discusses how the identity can be written if instead of the profit rate (r) one uses an estimated (hypothetical) rental rate of capital (ω), assumed to be the marginal product of capital. The measure of value added in constant prices is not the same as a physical measure of output and cannot be used interchangeably, notwithstanding the current practice. There is a parallel identity for total output (Y) including intermediate inputs (Z). This is:

[12] Samuelson (1979, 932) made it clear that identity (4) is correct. $P = rJ$ is the residually computed vector of profits, even though there is no reason to believe that noncompetitive industries have a common profit rate r and use capital J in proportion to $(V - wL)$. See also Simon (1979a).

$$Y \equiv W + P + Z \equiv wL + rJ + Z \quad (5)$$

Note that the real value of materials (Z) cannot be dichotomized into the product of a price times a quantity in our dataset because these variables only exist at a highly disaggregated level (e.g., quantities of electricity, water). This does not affect the argument insofar as the identity remains.

We carry the analysis on below with the value-added identity, but this follows through equally for gross output. This section is organized as follows. The argument is first introduced for cross-sectional data and then for time series. Although the arguments are conceptually the same, this allows a better understanding of why cross-sectional estimates of production functions (especially Cobb-Douglas) yield elasticities deemed plausible, while time-series estimates are much worse (including negative values at times). This is followed by a discussion of what truly causes the bias. The last subsection is a discussion of why the accounting identity argument does not apply when the series used are physical quantities.

3.1 Cross-sectional Data

To prove the identity argument, we start with cross-sectional data. Note that the factor shares in value added in a cross section can be written as $a_i = (w_i\, L_i)/V_i$ (employment) and $(1 - a_i) = (r_i\, J_i)/V_i$ (capital), where the subscript i refers to the cross-sectional unit. Note that these expressions are the identities (3)–(4) for a cross section, i.e., $V_i \equiv W_i + P_i \equiv w_i L_i + r_i J_i$, expressed as shares. For a low dispersion of the factors shares, the approximation $\bar{a} \simeq (\bar{w}\bar{L})/\bar{V}$, where a bar denotes the average value of a variable, holds. Then, the following also holds:

$$a_i/\bar{a} \simeq [(w_i L_i)/V_i]/[(\bar{w}\bar{L})/\bar{V}\,] \quad (6)$$

and

$$(1 - a_i)/(1 - \bar{a}) \simeq [(r_i J_i)/V_i]/[(\bar{r}\bar{J})/\bar{V}\,] \quad (7)$$

For small deviations of a variable X_i from its mean $\bar{X}$, it follows that $ln(X_i/\bar{X}\,) \simeq (X_i/X) - 1$. Taking logarithms of the previous two equations and using this approximation, it follows that:

$$ln[w_i/\overline{w}\ ] + ln[L_i/\overline{L}\ ] - ln[V_i/\overline{V}\ ] \simeq (a_i/\overline{a}) - 1 \quad (8)$$

and

$$ln[r_i/\bar{r}\ ] + ln[J_i/\bar{J}\ ] - ln[V_i/\overline{V}\ ] \simeq [(1 - a_i)/(1 - \overline{a})] - 1 \quad (9)$$

Multiplying equations (8) and (9) by $\overline{a}$ and $(1 - \overline{a})$, respectively, adding them, and rearranging the result yields:

$$\begin{aligned} lnV_i &\simeq B + \overline{a}lnw_i + (1 - \overline{a})lnr_i + \overline{a}lnL_i + (1 - \overline{a})lnJ_i = \\ &= B + lnM_i + \overline{a}lnL_i + (1 - \overline{a})lnJ_i \end{aligned} \quad (10)$$

where $B = (ln\overline{V} - \overline{a}ln\overline{w} - (1 - \overline{a})ln\bar{r} - \overline{a}ln\overline{L} - (1 - \overline{a})ln\bar{J})$ and $lnM_i = \overline{a}lnw_i + (1 - \overline{a})lnr_i$. The weighted average of the factor prices lnM_i plays a very important role in the discussion of the identity argument. Integrating:

$$V_i \simeq exp(B)\, w_i^{\overline{a}} r_i^{1-\overline{a}} L_i^{\overline{a}} J_i^{1-\overline{a}} \quad (11)$$

Equation (11), an approximation to the identity, will work well *if* the variation in factor prices within the cross section is not great, which is often the case in many empirical applications. Then $lnM(i)$ will be about constant; that is, $B + lnM_i$ in equations (10) and (11) will be about constant. This explains why Cobb-Douglas cross-sectional regressions like $V_i = CL_i^{\alpha} J_i^{\beta} e^{\varepsilon_i}$ tend to perform relatively well.

3.2 Time-series Data

Returning to the accounting identity now with time series, $V_t \equiv w_t L_t + r_t J_t$, where the subscript t denotes time, totally differentiate it with respect to time and express it in growth rates. This yields:

$$\hat{V}_t \equiv a_t \widehat{w}_t + (1 - a_t)\hat{r}_t + a_t \hat{L}_t + (1 - a_t)\hat{J}_t \quad (12)$$

where a circumflex over the variable denotes a growth rate, $a_t = W_t/V_t$ is the employment share in value added, and $(1 - a_t) = P_t/V_t$ is the capital share. Again, no assumptions have been made.

Now assume that factor shares happen to be constant across time.[13] Then the identity can be integrated to become:

$$V_t \equiv B w_t^a r_t^{1-a} L_t^a J_t^{1-a} \tag{13}$$

where $B = a^{-a}(1-a)^{-(1-a)}$ is the constant of integration, a is the (constant) employment share, and $(1 - a)$ is the (constant) capital share. Equation (13) is exact for each period, identical to $V_t \equiv w_t L_t + r_t J_t$. We stress that no assumption has been made, which means that the expression obtained is the original accounting identity (provided factor shares are constant). As above for cross-sectional data, the variable $M = w^a r^{1-a}$ (weighted average of the factor prices) is key to this discussion.

Given data for V, w, r, L, and J, then the value of V given by equation (13) will be identical to that given by $V_t \equiv w_t L_t + r_t J_t$ (and for being linked through an identity, their statistical properties, e.g., unit roots, are not the issue). The two equations are formally identical to each other. If over time the factor shares do not display significant variation, then the two equations for V will closely approximate each other.

Suppose now that in the unit under consideration, the weighted average of the wage and profit rates grows over time at a constant rate, i.e., $a\hat{w} + (1-a)\hat{r} = \varphi$. This means that equation (13) becomes:

$$V_t \equiv B e^{\varphi t} L_t^a J_t^{1-a} \tag{14}$$

[13] The assumption that factor shares are constant does not imply a Cobb-Douglas production function. Using simulation analysis, Fisher (1971) showed that if factor shares are constant, it is not because the economy's technology is Cobb-Douglas. Rather, it is the other way around; that is, the Cobb-Douglas form works empirically because factor shares happen to be constant (if they are). The latter could be simply due to the fact that firms use a constant markup on unit labor costs to set prices.

Equation (14) is still nothing more than the identity. It should be noted that it is not necessary for the time trend to be linear. This appears in equation (14) as a result of the assumption that the weighted average of the wage and profit rates grows over time at a constant rate. The regression $V_t = Ae^{\varphi t} L_t^{\alpha} J_t^{\beta} e^{\varepsilon_t}$ is often used in time-series estimations of putative production functions under the assumption that technical progress grows at the constant rate, φ. Certainly, this assumption can be changed as there is nothing in neoclassical economics that forces technical progress to grow at a constant rate. Results tend to be rather poor as a result of the fact that the linear time trend is not a good proxy for the weighted average of the wage and profit rates. In reality, the trend could take a more general mathematical form. In this case, then a more flexible time trend would have to be used to capture the path of the weighted average of the factor prices. For example, if the factor shares change secularly over time (due to, say, changes in the bargaining power of labor) and are hence correlated with the change in the capital–labor ratio, it is necessary to have a more flexible form to estimate the transformation of the accounting identity. One option is to use the Box-Cox estimation procedure, which can give a CES function, but the above interpretation is not affected by this. They are both merely alternative specifications of the identity.

The conclusion is that the estimation of a putative production function using constant-price value data cannot provide any evidence of the underlying technology of firms or sectors of the economy. [14]

A summary of the predictions of the derivations above is as follows:

[14] Econometricians like Cramer (1969, 234–37), Intriligator (1978), and Wallis (1979) also spelled out the argument. Yet they did not take it to its ultimate conclusion and did not test it. Intriligator (1978, 270) argued that the estimated elasticities would be *biased* toward constant returns to scale, and that they would be equal to the factor shares. Wallis (1979, 62–63) also concluded that the elasticities would be close to the factor shares and *tend* toward constant returns to scale, since the estimating equation is a close approximation to the accounting identity. This statement also misses the point. He proceeded to conclude that "perhaps the Cobb-Douglas results and the apparent support for constant returns or the marginal productivity theory are not as persuasive as was first supposed." Both assessments fall short to the extent that they do not clearly recognize that the identity makes the estimation a futile exercise: if one chooses the correct form, the factor elasticities must equal the factor shares always, hence they will add up to one, which may be confused with constant returns.

(i) The estimated factor elasticities must be equal to the factor shares, although this does not provide empirical evidence that factor markets are competitive.

(ii) As a consequence of the above, the estimated elasticities will add up to one, although this cannot be interpreted as evidence of constant returns to scale.

(iii) Estimation must indicate a very high fit (potentially one), independently of issues such as that the series may be random walks or display multicollinearity.

As noted earlier, neoclassical theory also has clear predictions about the above three (Solow 1974, 121; quoted above). The similarity between the predictions of the accounting argument and those of neoclassical theory can be misleading. The identity argument predicts that (i)-(ii)-(iii) cannot be rejected empirically *unless one fits the wrong functional form*, which is often the case. This is what creates the illusion that the theory is testable.[15]

This discussion has the following corollaries:

a) The identity $V \equiv wL + rJ$ can be transformed into equation (13) when factor shares are constant. However, it can be equally transformed into a multitude of specifications of the type $V \equiv f(M, L, J)$. The specific transformation to rewrite the identity as $V \equiv f(M, L, J)$ will depend on the actual paths of the factor shares, giving rise to forms that will look like, for example, CES or translog (i.e., the identity can be transformed into any form, not just the Cobb-Douglas). See appendix 4 for a discussion of the relationship between the CES function and the accounting identity.

b) One important reason that underlies the recent literature on the estimation of production functions is the desire to obtain an estimate of total factor productivity in levels (TFP) or growth rates ($\widehat{TFP}$), thought to be unobservable to the researcher, or a

[15] The exchange found across Shaikh (1974), Solow (1974), and Shaikh (1980) exemplifies this (worth reading). Shaikh (1974) pointed out that all Solow (1957) did was to reproduce the accounting identity. To prove Shaikh (1974) wrong, Solow (1974) fitted the regression $V_t = Ae^{\varphi t} L_t^{\alpha} J_t^{\beta} e^{\varepsilon_t}$ to his dataset. Results were poor, which led him to conclude that Shaikh (1974) was wrong. However, Shaikh (1980) showed that Solow's (1974) regression was flawed. The reason is that he fitted the wrong functional form to his own dataset, in particular the approximation of the weighted average of the wage and profit rates through a linear time trend. Instead, Shaikh (1980) fitted a Cobb-Douglas function but, instead of the time trend, he added a trigonometric function that captured well the weight average. This gave excellent results, i.e., very close to the identity. See also Solow (1987) and Shaikh (1987).

measure of our ignorance (see equation [3]). Its calculation requires estimates of the factor elasticities (α and β) to derive $lnTFP_t = lnV_t - \alpha lnL_t - \beta lnJ_t$ (in levels and assuming constant elasticities) or $\widehat{TFP}_t = \hat{V}_t - \alpha\hat{L}_t - \beta\hat{J}_t$ (in growth rates and assuming constant elasticities). However, the derivation above from the identity demonstrates that the best possible statistical fit of the regression will be the one where $\alpha = a$ and $\beta = (1-a)$, i.e., elasticities must be equal to the factor shares, unless the approximation to the identity is incorrect. This implies that $lnTFP_t \equiv lnV_t - lnB - alnL_t - (1-a)lnJ_t \equiv alnw_t + (1-a)lnr_t$ (in levels) and $\widehat{TPF}_t \equiv \hat{V}_t - a\hat{L}_t - (1-a)\hat{J}_t \equiv a\hat{w}_t + (1-a)\hat{r}_t$ (in growth rates). These equations are correct by definition and hence always true. This is the tautologically correct value of total factor productivity (growth).[16]

c) The methods discussed in the literature to overcome the endogeneity problem, if they are satisfactory in the sense that they deliver plausible estimates of the elasticities (i.e., close to the factor shares) it is because, by serendipity, they manage to track well the accounting identity or some mathematical transformation of it.

3.3 The "Bias"

Given the arguments above, the likely deviation of the elasticities α and β from the factor shares in, for example, the regression $V = AL^{\alpha}J^{\beta}e^{\varepsilon}$ is not the result of a true endogeneity problem (in the econometric sense) but of omitting $M = w^{a}r^{1-a}$. This is akin to omitted-variable bias but with two important differences. First, in this case we know that what has been omitted is M. Second, the "true model" is the complete accounting identity $V \equiv Bw^{a}r^{1-a}L^{a}J^{1-a}$, which contains no random error term. The error in the estimated equation is M.

Treated as if it were an omitted-variables problem, it is straightforward to derive the expected values of the estimated coefficients ($\hat{\alpha}_{OLS}$ and $\hat{\beta}_{OLS}$). To do so, substitute the "true model" (i.e.,

[16] Barro's (1999) suggestions to improve growth accounting require reconsideration in light of this analysis: (i) deal with the problem of regressors' endogeneity: it is not the problem if the regression is run using constant-price value data; (ii) capital is measured with errors: correct but not the issue if the regression is run using constant-price value data; and (iii) the need to allow for variations in factor shares and in the growth of TFP: correct. When this is done, one will find the identity.

the accounting identity equation [13]) into the OLS estimates of α and β from $V = AL^{\alpha}J^{\beta}e^{u}$, and take expectations. The result is:

$$E[\hat{\alpha}_{OLS}] = a + \Psi = a + E\left[\frac{Var(j)Cov(\ell,\mu) - Cov(j,\mu)Cov(\ell,j)}{Var(\ell)Var(j) - [Cov(\ell,j)]^2}\right] \tag{15}$$

and

$$E\big[\hat{\beta}_{OLS}\big] = (1-a) + \Phi = (1-a) + E\left[\frac{Var(\ell)Cov(j,\mu) - Cov(\ell,\mu)Cov(\ell,j)}{Var(\ell)Var(j) - [Cov(\ell,j)]^2}\right] \tag{16}$$

where lowercase letters denote the logarithms of the corresponding uppercase letters in the text, i.e., $\ell = lnL$, $j = lnJ$, $\mu = lnM = alnw + (1-a)lnr$. Equations (15)–(16) indicate that the (OLS) expected values of α and β will be the employment and capital shares, a and $(1-a)$, respectively, plus the terms Ψ and Φ (the "bias"). The last two capture the covariation between L, J, and the omitted but known term M. The important point to note is that the first term of the two expected values is the corresponding factor share, not just any undefined coefficient. It can be seen that Ψ and Φ are not econometric biases that result from regressors' endogeneity but merely the outcome of omitting M.

This bias would tend to decrease if the omitted variable M could be well approximated by a variable highly correlated with it. As one better approximates the omitted variable, then Ψ and Φ will become smaller.[17] This variable could be one of those that often appears in growth regressions, e.g., human capital, which at times improves the goodness of fit. A second option is to construct a trigonometric function, or a high-order polynomial, that tracks M. Finally, a third option would be to correct the capital stock for changes in capacity utilization. This will increase this variable's cyclical fluctuation and reduce that of r, and hence M, if factor shares are roughly constant (M tends to be procyclical, caused largely by the cyclical variation in the profit rate). The goodness of fit will increase, and coefficients will consequently approximate the factor shares. In

[17] Naturally, inclusion of M (or of a variable perfectly correlated with M) in the regression makes $\Psi = 0$ and $\Phi = 0$, i.e., there is no bias.

any case, all that has been done is to approximate the accounting identity more closely.[18] Clearly, this is not a problem about the choice of estimation method.

3.4 Production Functions with Physical Quantities

The problem discussed above would not happen if the researcher had true physical data (Y^*, L, K, Z^*) because these are not linked through an accounting identity like the one with value data. Now $Y^*$is total output and not value added (there is no value added in physical terms) and, therefore, the production function has to include materials Z^*.[19] Indeed, with physical quantities it would be possible to estimate the production function and obtain the true elasticities, which may differ from the factor shares.

Suppose there exists a production process that converts L (number of workers), K (number of identical machines), and Z^*(kilowatts of electricity) into Y^*(widgets). Assume that this production process is given by $Y^* = AL^{\alpha}K^{\beta}Z^{*(1-\alpha-\beta)}e^{\varepsilon}$. This specific form is not essential to the argument. As this is an "engineering" or physical production function, output must be determined by the correctly measured flow of services from labor and capital and the rate of utilization of materials.

Now a researcher estimates $lnY^* = d + b_1 lnL + b_2 lnK + b_3 lnZ^* + \varepsilon$. What would the estimates of b_1, b_2, and b_3 pick up? We argue that she would obtain the true technological relationship, i.e., α, β, and $(1-\alpha-\beta)$. In this case, the series Y^*, L, K, Z^* are not definitionally related through an accounting identity. Notice though that one could construct an infinite number of accounting identities with arbitrarily chosen weights (b, c) that would determine the distribution of factor rewards in physical terms, e.g., $v = b(Y^*/L)$, $x = c(Y^*/K)$, and $p = (1-b-c)(Y^*/Z^*)$, with $0 < b < 1$ and $0 < c < 1$, and then construct $Y^* \equiv vL + xK + pZ^*$, i.e., the identity in physical terms. This expression could then be transformed into the one that resembles the production

[18] There is a literature on the omitted-price bias (Van Beveren 2012, 102–4). This acknowledges that since firm-level prices do not exist, researchers use industry-level deflators. They use the resulting figures as proxies for quantities. The literature acknowledges that this may result in, for example, significant biases in estimated TFP (Foster, Haltiwanger, and Syverson 2008). First, this literature fails to acknowledge the accounting identity problem when researchers use constant-price data. Second, even when authors use microeconomic data, the physical quantities needed to properly estimate TFP do not exist, in particular, the capital stock.

[19] Note that in equation (1) we used Q to refer to physical output in a value-added production function. This is clearly wrong. We did so to follow the convention.

function. The important point now is that there are no actual statistics relating Y^*, L, K, Z^*. It is for this reason that the regression will pick up the true elasticities and not the factor shares—an infinite number, depending on the values of (b, c). However, with value data, the series of output (Y), inputs (L, J, Z), and the factor shares $a^* = wL/Y$, $b^* = rJ/Y$ and $(1 - a^* - b^*) = Z/Y$ are related through only one identity. This is the identity that the monetary data regression will undoubtedly pick up.[20]

Finally, it must be noted that with physical quantities, researchers face two problems. First is that one would need to know what functional form to estimate. The second problem is that, in practice, estimating a production function for manufacturing (or services) with physical quantities is next to impossible. This is due to the data requirements needed (e.g., individual capital stocks for an oil refinery). If these two issues were addressed, then the endogeneity of the regressors would be a correct concern because there is no identity directly linking Y^*, L, K, and Z^*, and the regression would contain a true econometric error.

4. UNDERSTANDING "PRODUCTION FUNCTION" ESTIMATIONS THROUGH THE LENS OF THE ACCOUNTING IDENTITY

Table 2 shows estimation results of the accounting identity written as a Cobb-Douglas production function, with value added (V) as the dependent variable, using the same five estimators as in table 1: ordinary least squares (OLS), least-squares dummy variables (LSDV), instrumental variables (IV), system generalized method of moments (GMM), and Levinsohn and Petrin (L-P).

[20] It is easy to design and simulate the following experiment. Suppose the researcher has physical data (Q, L, K) to estimate the elasticities of the true production function (assume it is Cobb-Douglas) for i firms. The elasticities are, by design, 0.25 for employment and 0.75 for capital (with a random error to avoid multicollinearity). K is the number of identical machines. The profit rate is the same across firms, $r_i = r = 0.10$. Now, assume firms set prices by applying a markup (μ) on unit labor costs, i.e., $p_i = (1 + \mu)(w_i L_i / Q_i)$, where $\mu = 0.33$. The money wage rate is the same across firms, i.e., $w_i = w$. This mechanism is used to generate the price (p), and then output in monetary terms $(V_i = p_i Q_i)$, the shares $\alpha_i = \left(\frac{w_i L_i}{V_i}\right) = \left(\frac{1}{1+\mu}\right) = 0.75$ and $1 - \alpha_i = \left(\frac{r_i J_i}{V_i}\right) = \left(\frac{\mu}{1+\mu}\right) = 0.25$, and, therefore, the accounting identity $V_i \equiv w_i L_i + r_i J_i$ (actually, the identity is generated as $J_i \equiv (V_i - w_i L_i)/r_i$ because J is generated residually). Now run the regression $V = f(L, J)$ (also Cobb-Douglas). The estimated "elasticities" will undoubtedly be 0.75 for employment and 0.25 for capital, and not the other way around.

The arguments in section 3 and the results in table 2 help one to understand the results in table 1. Appendix 2 (tables A2 and A3) shows the results (standard Cobb-Douglas and identity written as a Cobb-Douglas) but with gross or total output (Y) as the dependent variable. The main results to highlight are, first, that the best possible results one can obtain estimating a production function with monetary values are those embodied in the accounting identity (which has no error term), i.e., estimated elasticities will equal the factor shares. Second, the estimation method does not matter.

Given the arguments in section 3, it was necessary to find a dataset that would allow constructing the accounting identities for value added and total output. This is required to get the factor prices of employment (i.e., wage rate) and of the capital stock (i.e., the profit rate). Details are provided in appendix 1. Appendix 3 also shows how the accounting identity was reconstructed with a hypothetical rental price of capital. The Cobb-Douglas function is reestimated with this alternative dataset with the five estimators. It is shown that qualitatively this does not affect the argument.

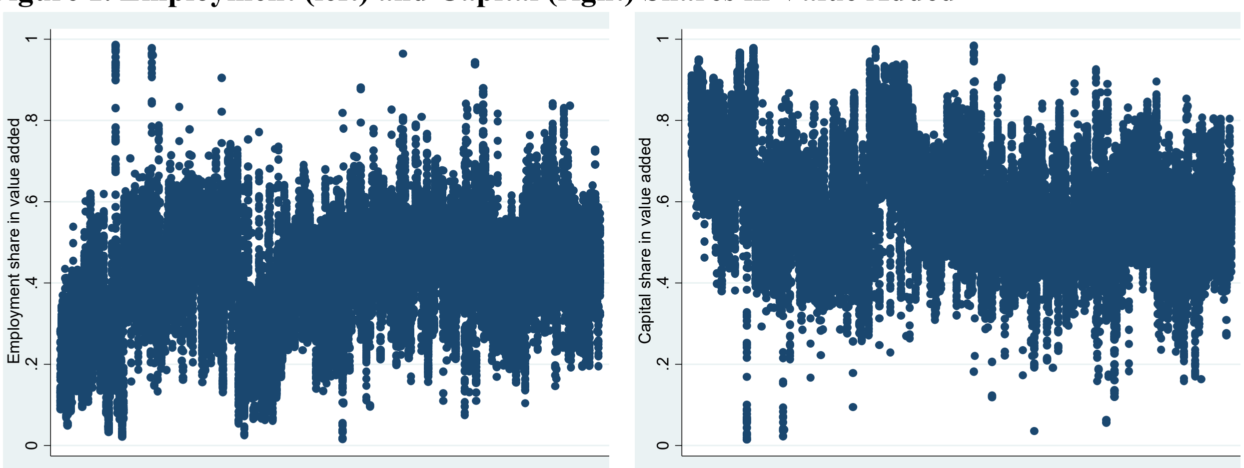
We estimated the Cobb-Douglas in accounting identity form (i.e., including the factor returns as regressors) using the same five estimation methods. Estimation can be undertaken because factor shares are not exactly constant across time and sectors. Otherwise, the variables in the identity would be exactly related and this would cause perfect multicollinearity. The identity derived with cross-sectional data is equation (11), and that derived for time-series data is equation (13). It is important to keep this in mind because both dimensions are pooled. Recall that these equations were derived in section 3 under the assumption that wage and profit rates vary little within each cross section (for cross-sectional data) and that factor shares vary little across years (for time-series data). It is also important to emphasize that there are many more sectors (473) than years (54).[21] The equations estimated now add the wage and profit rate as regressors. It is then important to understand the purpose of this exercise: the claim is not that a production function

[21] The variable $lnM_i = \bar{a}lnw_i + (1-\bar{a})lnr_i$ was constructed for all 54 years and corroborated that indeed, in practically all cases, the variable varies within a small range. Hence the Cobb-Douglas form $V_i = CL_i^{\alpha}J_i^{\beta}e^{\varepsilon_i}$ has to give a close statistical fit. On the other hand, when the time-series regressions $V_t \equiv Ae^{\varphi t}L_t^{\alpha}J_t^{\beta}e^{\varepsilon_t}$ are run for the 473 industries, results are very poor in most cases, yielding negative or above-one elasticities, and statistically insignificant. This corroborates that the linear time trend in time-series regressions is a bad proxy for the weighted average of the wage and profit rates. Results are available upon request.

should be estimated including the factor prices as explanatory variables.[22] This is done solely to compare the results obtained in table 1 with those when the full identity is estimated (table 2), and to show that as the regression fit improves, the estimated elasticities approach the factor shares. The estimation method becomes a secondary issue.

For reference, the average employment and capital shares in value added across the entire dataset are 0.41 and 0.59, respectively. The shares are graphed in figure 1 for all industry codes (horizontal axis) and all years. They show significant variation, including values (both high and low) that seem to be far from the mean. A two-way ANOVA test rejects the null hypotheses that the means across industries are equal and that the means across years are equal.

Figure 1. Employment (left) and Capital (right) Shares in Value Added



Source: Authors.
Note: Horizontal axes are the 473 industry codes

The results in columns (1), (2), (4), (5), and (6) in table 2 indicate that the coefficients of the wage rate and employment, and of the profit rate and capital stock, are close to each other across all estimation methods. This is as expected, based on identities (11) and (13). Column (3) is discussed below. Moreover, they are not far from the respective factor shares, despite the variation reported in figure 1; hence they add up to one. In all cases, the R-squared is almost one. Predictions (i)-(i)-(iii) in section 3.2 seem to be validated.

[22] The full identity (equation [11] or [13]) including (w, r) might be somewhat reminiscent of Griliches and Mairesse's (1998) suggestion to use input prices are instruments. Yet, this is not what the identity argument is about.

Table 2. The Accounting Identity: Dependent Variable is Value Added (lnV)

	OLS	LSDV (sector dummies)	LSDV (wage-profit dummies)	IV[23]	System GMM[24]	L-P[25]
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Constant	0.70 (57.8)	0.91 (44.5)	1.58 (16.3)	0.94 (47.1)	0.67 (3.24)	
Value added lagged (V_{t-1})					0.11 (5.86)	
Wage rate (w)	0.38 (247)	0.43 (222)		0.42 (225)	0.47 (20.5)	0.39 (25.0)
Profit rate (r)	0.61 (593)	0.57 (429)		0.58 (448)	0.47 (16.4)	0.61 (35.7)
Employment (L)	0.38 (416)	0.41 (308)	0.28 (38.2)	0.40 (318)	0.42 (22.0)	0.38 (34.0)
Capital (J)	0.62 (714)	0.58 (478)	0.72 (115)	0.58 (492)	0.52 (30.3)	0.68 (23.9)
Sector fixed effects		Yes	Wage/profit dummies	Yes	Yes	
R^2	0.995	0.998	0.998	0.998	N.A.	N.A.
Degree of returns to scale	0.998 (2,000)	0.988 (944)	0.999 (255)	0.987 (962)	0.937 (113)	1.056 (44.9)
No. of observations	25,305	25,305	473 Year: 2011	24,826	24,826	25,305

Source: Authors

Notes: (i) Regressors are expressed in logarithm; (ii) columns (1)–(3): t-values are in parentheses; columns (4)–(6): z-values are in parentheses; (iii) columns (1)–(6): z-values are in parentheses under degrees of returns to scale; (iv) column (5) shows the long-run elasticities of employment and capital (z-values in parenthesis). If the GMM system is estimated without the lagged dependent variable ($ln\, V_{it-1}$), results are similar.

The values of the elasticities shown in table 1 and table 2 help one understand the theoretical discussion in section 2.[26] This means that the discrepancy between the elasticities and the factor shares in table 1 is due to the "bias" that results from the omission of the factor prices (wage and profit rates) and it is not caused by regressors' endogeneity (as discussed in section 3.3). As noted

[23] Estimation as in table 1. Results reported use capital lagged one period as the instrumental variable. Estimation is two-stage least squares. Since what is being estimated is an accounting identity, tests for the validity of the instrument do not apply.

[24] Estimation as in table 1. Since what is being estimated is an accounting identity (or an equation very close to it), the tests for autocorrelation and validity of the instruments do not apply.

[25] Estimation as in table 1. Wage and profit rates are assumed to be freely variables, like labor.

[26] As expected, the Durbin-Wu-Hausman test now does not reject the null hypothesis of no endogeneity, i.e., that the error term is not correlated with the regressors, as adding the residual of the regression of the potentially endogenous variables does not add information to the identity.

above, the estimation method is a secondary issue: the more binding the exogeneity or moment conditions imposed in an effort to deal with endogeneity are (i.e., the more variation they force us to discard), the more the remaining variation deviates from the accounting identity and the less sensible the coefficients will become.

The five methods used yield slightly different estimates as a result of how each of them approximates the identity, but the conclusion is clear, namely, that in all cases the identity determines the results obtained (elasticities). OLS seemed to yield the most sensible results in table 1 because, overall, it is the method that provides the simplest solution to the true problem, namely the omission of the weighted average of the factor prices $M_{it} = {w_{it}}^{a} r_{it}^{1-a}$. Given that the latter does not show a great variation, the best option, relative to what the other methods do, is to subsume this variable into the constant term.

Finally, for being virtually the accounting identity, it makes little difference to the coefficients if these regressions are estimated with the variables in growth rates instead of levels (results available upon request).

To complete the analysis of the identity, appendix 4 (figures A2 and A3) provides time-varying estimates (Kalman filter) of the accounting identity in growth rates, equation (12), for a sector (pure times series). Results indicate that the four estimated coefficients are indeed the factor shares, which add up to one (with minimal variation) every single period. Appendix 4 also offers a discussion of the CES production function and the identity because factor shares in the sector estimated are not constant.

4.1 A Reexamination of the Results in Light of the Identity

The two main results of the discussion so far are, first, that the estimated coefficients must be the factor shares and, second, that the main problem is not the endogeneity of the regressors but the bias caused by omitting the wage and profit rates. To the extent that these variables are not adequately proxied through other variables included in the regression, the coefficients of employment and capital (and materials) will be biased estimates of the average factor shares.

The focus next is on the LSDV and L-P methods, and on the implications of the identity argument for TFP.

4.1.1 The LSDV Method

This omits the wage and profit rates and adds sectoral dummies. Comparing the results in column (2) in table 1 and table 2, the conclusion is clear: the sectoral dummies in table 1 do not pick up the weighted average of the wage and profit rates well, with the consequence that the coefficient of capital in the value-added regression is very high, i.e., far from the average share (0.79 in table 1). The same applies to table A2 (Cobb-Douglas) and table A3 (identity) for gross output (see appendix 2).

To see how the identity argument helps solve this problem, the 473 sectors were ranked according to the value of the weighted average of the wage and profit rate. This was done for each year individually, i.e., $lnM_i = \bar{a}lnw_i + (1 - \bar{a})lnr_i$, where the average employment and capital shares $\bar{a}$ and $(1 - \bar{a})$ are calculated for each period across all the sectors. We divided lnM_i into 200 homogeneous groups from smallest to largest and constructed group dummy variables (D_m) for each of them (without the sector dummies σ_i). Then the following regression was estimated:

$$lnV_i = \pi_0 + \alpha lnL_i + \beta lnJ_i + D_m + \varepsilon_i \quad (17)$$

This procedure yields results much closer to the accounting identity, with α and β approximating the corresponding factor shares (and also in the case of total output). The results are shown in column (3) in table 2 for value added for 2011, with coefficients 0.28 (employment) and 0.72 (capital) (the average employment and capital shares are 0.31 and 0.69, respectively), and in column (3) in table A3 in appendix 2 for gross output for 2011, with coefficients 0.12 (employment), 0.34 (capital), and 0.54 (materials) (the average employment, capital, and materials shares are 0.15, 0.33, and 0.52, respectively). While we have shown the results in the table for only one year, this was done for every single year (constructing the dummies appropriately) and in all cases the estimated elasticities are very close to the average factor share of the corresponding year (results available upon request).[27] Consequently, the LSDV estimates in table 1 differ from the shares not because of endogeneity but because the dummies do not capture lnM_i well.

4.2.2 The L-P Method

This starts with an OLS regression that contains a third-degree polynomial in capital and materials. The estimate of the elasticity of employment is obtained in this first step; it then adds a second step to search for the value of β (the elasticity of capital). This second step minimizes the sample residual: $\min_{\beta^*} \sum_{it}\left(ln\, V_{it} - \hat{\alpha}\, ln\, L_{it} - \beta^*\, ln\, J_{it} - E\left[\theta_{\iota t}\widehat{|\theta_{\iota t-1}}\right]\right)^2$, where $\hat{\alpha}$ is the employment elasticity obtained in the first step, θ is the transmitted productivity shock, and $E\left[\theta_{\iota t}\widehat{|\theta_{\iota t-1}}\right]$ is a function of the residuals in the first step.

However, the accounting identity (see equations [11] or [13]) implies that:

$$\Omega_{it} \equiv (lnV_{it} - lnB - alnL_{it} - (1-a)lnJ_{it} - \mu_{it}) = 0 \qquad (18)$$

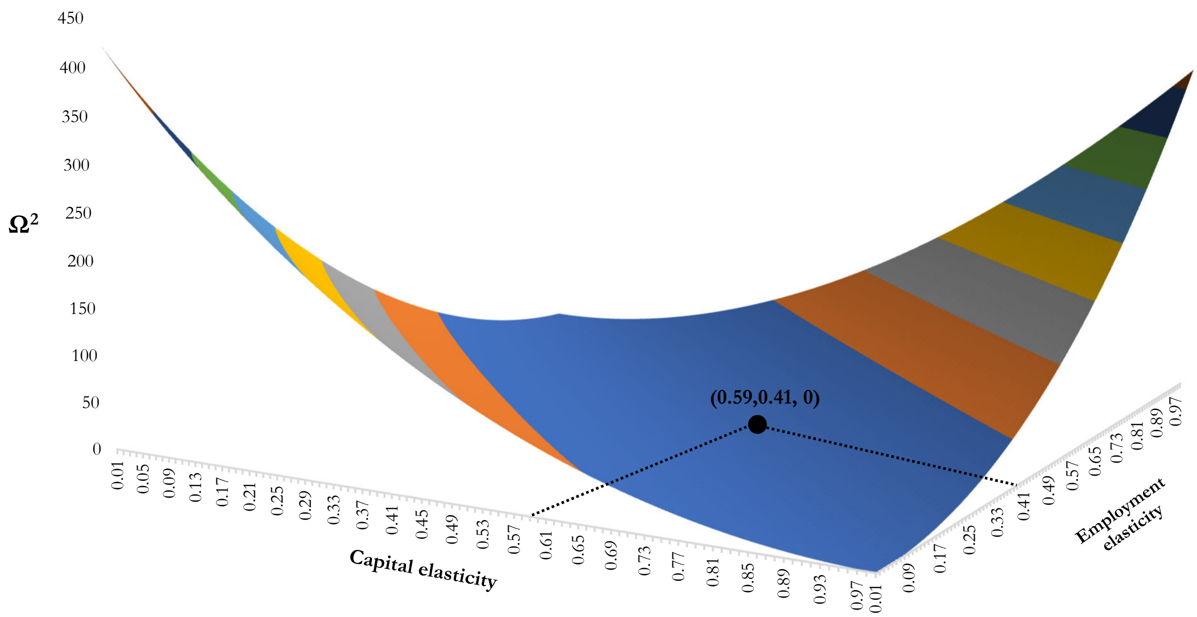
where $\mu_{it} = alnw_{it} + (1-a)lnr_{it}$ and $B = a^{-a}(1-a)^{-(1-a)}$. What this means is that the values of the elasticities of employment and capital that minimize Ω (more precisely, make it equal to zero) are the factor shares. This is exact for each unit (sector) and time period. Figure 2 graphs, for one sector and one year, combinations of employment and capital elasticities (from 0.01 to 0.99) and Ω^2 (in the vertical axis). The latter becomes zero at precisely the values of the employment and capital shares, $\alpha = a = 0.41$ and $\beta = (1-a) = 0.59$, respectively.[28]

Three reasons explain why the estimates produced by the L-P method in table 2 may not exactly equal the average factor shares. First is that the employment elasticity obtained in the first step may not be accurately estimated (i.e., far from the employment share). Second, the search process in the second step may in fact distort the result to the extent that $E\left[\theta_{\iota t}\widehat{|\theta_{\iota t-1}}\right]$ may differ

[27] Naturally, if all years are pooled, results deteriorate as the average factor shares change across time. Likewise, results are poor (i.e., elasticities differ significantly from the factor shares) if each individual cross-sectional regression is estimated with the sectoral dummies instead of with the wage-profit dummies that we have created.
[28] Naturally, combinations of the elasticities close to the factor shares give values of Ω^2 close to zero.

significantly from μ. Third, the estimation process yields a single estimate point for the elasticities (factor shares) when in reality factor shares vary somewhat across sectors and time.

Figure 2. Employment and Capital Elasticities and Ω^2



Source: Authors
Notes: $\Omega_i \equiv (lnV_i - lnB - \alpha lnL_i - \beta lnJ_i - \alpha lnw_i - \beta lnr_i)$; $lnB = \ln(\alpha^{-\alpha}\beta^{-\beta})$
Industry code is 334310 (Audio and Video Equipment Manufacturing), year 1975

Further evidence that the accounting identity drives the results is provided by the fact that when the L-P procedure (full identity) is run with capital as the freely variable input in the first step (together with the wage and profit rates, and employment as the state variable), results in row (1) in table 3 do not change qualitatively with respect to those shown in column (6) in table 2. Yet, if the L-P procedure is run as in table 1, column (5) for the Cobb-Douglas putative production function, results continue being very poor, as shown in table 3, row (2).[29] Reversing the roles of capital and labor is the converse of the assumption that is normally made. The standard assumption (capital as the state variable) is that, in the face of an exogenous shock, firms optimize their capital stock, presumably by altering their investment decisions rather than by adjusting their

[29] This exercise has been repeated for the gross output regression and obtained the same qualitative results; that is, reversing the variables in the identity makes no difference. Results are available upon request.

labor input. However, it can be equally argued that, as the flow of capital services is a sunk cost, the immediate response may be to reduce employees' hours of work and introduce a furlough scheme and layoffs. Thus, in these circumstances, employment is the state variable. The reason why the choice of the state variable makes little difference once wage and profit rates are included is that the results are being driven by the underlying account identity.

Table 3. Levinsohn-Petrin (L-P) Method with Employment as State Variable: Cobb-Douglas and Accounting Identity (dependent variable: lnV)

	Wage rate (w)	Profit rate (r)	Employment (L)	Capital (J)	Degree of returns to scale	No. of observations
L-P Identity	0.41 (25.6)	0.61 (30.5)	0.41 (21.3)	0.61 (36.5)	1.017 (132)	25,305
L-P Cobb-Douglas			0.83 (25.9)	0.31 (21.2)	1.135 (14.5)	25,305

Source: Authors.
Notes: (i) Regressors are expressed in logarithm; (ii) z-values are in parentheses; (iii) column (2): chi-squared (Wald test) to test the null hypothesis that the degree of returns to scale is 1 is in parenthesis; (iv) the freely variables in column (1) are capital (J), wage rate (w), and profit rate (r). The freely variable in column (2) is capital (J).

4.2.3 TFP

We showed in section 3 that the residually calculated TFP (as the difference between output and weighted inputs) is, definitionally, the weighted average of the wage and profit rates, and hence this residual is, contrary to what the literature claims, observable to the researcher. Algebraically:

$$lnTFP_{it} \equiv (lnV_{it} - lnB - alnL_{it} - (1-a)lnJ_{it}) \equiv (alnw_{it} + (1-a)lnr_{it}) \qquad (19)$$

where the weights are, once again, the corresponding factor shares.

Figure 3 (left-hand side) shows TFP for one sector and 54 years, with the initial year equal to one. Figure 3 (right-hand side) shows *TFP* for one year and all sectors, in this case as a ratio of the highest value. Given that V and J are measured in dollars (euro, yen, etc.) and L is measured in number of workers, then TFP is unitless, an index. This has the same value if it is calculated as the weighted average of the wage rate (w) and the profit rate (r). This is so simply by virtue of the identity.

This raises two issues. The first one, a lesser point, is that the absolute level of calculated TFP is dependent on the units of the variables.[30] Second, and more fundamental, is the question of the interpretation of TFP, given that here it has been derived solely from the distribution accounting identity and not from a production function, i.e., no theory. In neoclassical production theory, the growth of the residually calculated TFP captures the shift in the "true" production function (i.e., with physical quantities) keeping inputs constant. This is what the theory is about and the reason for empirically estimating (testing) the production function. In the literature, it is referred to as the *primal* measure. However, this calculation (i.e., the residual) does not have any clear interpretation when looked upon strictly from the accounting identity (because it has not been derived from a production function). What this indicates is that this "residual" is numerically identical to the (growth rate of the) weighted average of the wage and profit rates. While this is correct, the interpretation in terms of productivity would require assuming that the wage rate is the marginal product of labor and the profit rate is the marginal product of capital, i.e., a production function. This is circular reasoning, apart from the fact that $\partial V/\partial L = w$ and $\partial V/\partial J = r$ are tautologically true in the identity.

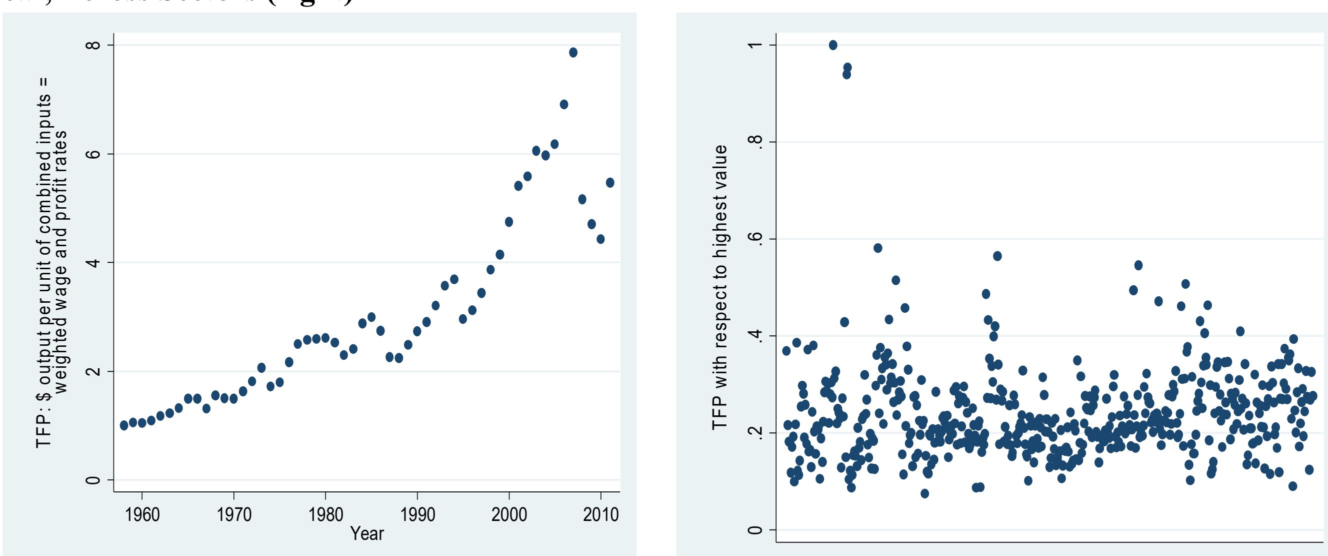
Finally, we add that if output and inputs were measured in physical quantities, there would be no residual. This was pointed out by Jorgenson and Griliches (1967, 249) referring explicitly to a physical production function in terms of only the inputs L and K: "In the terminology of the theory of production, if quantities of output and input are measured accurately, growth in total output is *largely* explained by growth in total input" (italics added). The only difference with our interpretation is that "largely" should be "entirely," i.e., no residual.

[30] If, for example, employment was measured as number of hours instead of number of workers, the level of TFP would be different. Naturally, if this were the case, the units of the wage rate would be dollars per hour to preserve the accounting identity. Likewise, if the profit rate were expressed as 10 instead of 0.1, the capital stock would have to be divided by 100 to also preserve the identity. This means that the value of $TFP = V/(L^a J^{1-a})$ or of $TFP = w^a r^{1-a}$ would change. Hence, one cannot compare absolute levels. What remain unchanged, however, are the ratios with respect to the first year in the time-series case (as well as the growth rates), and with respect to the highest value for a cross section (if one uses common factor shares, e.g., average).

5. CONCLUSIONS

This paper considered the problem of regressors' endogeneity in production functions. It reported Cobb-Douglas estimates using five estimation methods. Using a dataset that covers 473 sectors (industries) for 54 years, it has been concluded that it is difficult to ascertain which method provides the correct elasticities of the factors of production. Somewhat paradoxically, simple OLS provides very sensible results. Theoretically more appropriate estimators to deal with the problem of endogeneity like GMM or L-P do not seem to yield better results. All methods tend to produce high statistical fits and, when results seem to be sensible, elasticities are not far from the factor shares, adding up to about one.

Figure 3. Total Factor Productivity (*TFP*) Index: One Sector, Time Series (left) and One Year, Across Sectors (right)



Source: Authors
Notes: One sector, time series (left):
Industry code = 334310 (Audio and Video Equipment Manufacturing)
Time: 1958-2011
$TFP_t \equiv exp[(lnV_t - lnB - \bar{a}lnL_t - (1-\bar{a})lnJ_t)] \equiv exp(\bar{a}lnw_t + (1-\bar{a})lnr_t);$
$lnB = ln(\bar{a}^{-\bar{a}}(1-\bar{a})^{-(1-\bar{a})})$
$\bar{a}$ is the average employment share (1958-2011), 0.38; $(1-\bar{a})$ is the average capital share (1958-2011), 0.62.
One year, across sectors (right):
$TFP_i \equiv exp[(lnV_i - lnB - \bar{a}lnL_i - (1-\bar{a})lnJ_i)] \equiv exp(\bar{a}lnw_i + (1-\bar{a})lnr_i)$
$lnB = ln(\bar{a}^{-\bar{a}}(1-\bar{a})^{-(1-\bar{a})})$
$\bar{a}$ is the average employment share (473 sectors), 0.35; $(1-\bar{a})$ is the average capital share (473 sectors), 0.65.
Year is 1997, 473 sectors
Industry code 311930 (Flavoring Syrup and Concentrate Manufacturing) has the highest value of TFP. TFP of this year is set equal to 1.

However, in retrospect, these results should not come as a surprise. It has been shown that the good statistical fit that many studies find is due to the fact that they use constant price value (monetary) data for output and capital (and materials in total output regressions), not physical data. The use of value data implies that the variables used in production function estimations are related through a distribution accounting identity. Hence, all that is estimated is a transformation of this accounting identity, which holds by definition, and it does not have an error term. This explains the generally high statistical fits found. Moreover, as a consequence of the identity, the estimated putative elasticities of employment, capital, and materials must, by definition, equal the factor shares, subject to the qualifications mentioned in the text, i.e., that the incorrect approximation (or omission) of the weighted average of the wage and profit rates yields other coefficients, and researchers believe there an endogeneity problem.

These arguments have been confirmed by the regression results reported in this paper. Ironically, the problem is not whether estimation of putative production functions using value data are subject to endogeneity bias. The problem is that it makes no sense to estimate them in the first place. This problem has no econometric solution, and the development of new alternative estimators will not yield new insights. The estimators/procedures devised by those researchers trying to correctly identify the elasticities of the production function would be valid with physical data, although as we have argued, the estimation with physical data poses other problems.

Finally, and as a consequence of the identity, we argued that estimated (with value added) total factor productivity is not a "measure of our ignorance." It is simply definitionally equal to the weighted average of the wage and profit rates, derived from the accounting identity.

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APPENDIX 1: DATA AND VARIABLES

We used the data on output, payroll, employment, real capital stock, materials cost, and price indices from the Manufacturing Industry Database[31] of the National Bureau of Economic Research (NBER) and the US Census Bureau's Center for Economic Studies (CES). The database contains annual data from 1958 to 2011 for 473 industries (based on six-digit 1997 North American Industry Classification System, subsectors 31-32-33). The dataset contains 25,542 observations.[32] There are many more sectors (473) than years (54). All series are in constant prices. Given that the key to the argument is the accounting identity, the fact that we use industry-level data and no firm-level data is not a problem. As we mentioned, the identity holds at all levels (firm, industry, economy).

This database contains all the necessary variables to construct the total output (Y) identity $Y \equiv W + P + Z \equiv wL + rJ + Z$. W and P are as defined earlier; Z represents the real value of the total cost of materials. Real value added is $V \equiv W + P$. The variables L, J, and Z are those used to estimate the production functions and the identities. Appendix 3 shows how to construct the identity with a hypothetical rental price of capital.

We constructed the accounting identity for real total output (Y) using the real (deflated) values of (variable names as in the NBER database): real total output $\left(\frac{vship}{piship}\right)$ = real total wage bill $\left(\frac{pay}{piship}\right)$ + real total profits + real total materials cost $\left(\frac{matcost}{piship}\right)$. *vship* is the nominal total value of shipments, *pay* is the nominal total payroll, *matcost* is nominal total cost of materials, and *piship* is the price deflator of shipments. Real total profits (P) is estimated residually to ensure that the identity holds. Real total output (Y) is in US dollars and so are the real total wage bill (W), real total profits (P), and real cost of total materials (Z).

The real average wage rate (w) was calculated as $\left(\frac{W}{emp}\right)$, where W is the real total wage bill and *emp* is the total employment.

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[31] Available at: https://www.nber.org/research/data/nber-ces-manufacturing-industry-database

[32] The dataset is an unbalanced panel. Not all industries cover 54 years.

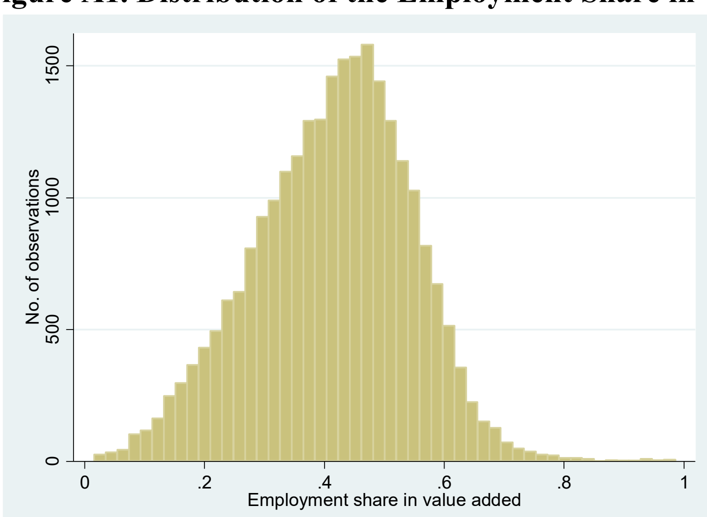
The real profit rate (r) was calculated as $\left(\frac{P}{cap}\right)$, where P is real total profits and *cap* is the dollar value of the total real capital stock (J).

The above ensures that $Y \equiv W + P + Z \equiv wL + rJ + Z$ holds and it does not depend on any theory (Samuelson 1979, 932; Simon 1979a).

It is important to note that the calculated profit rates range from -243 percent to 3,734 percent. Our calculations indicate that there are 80 observations with negative profit rates and 6,715 above 100 percent. The dataset contains 25,542 observations, covering 473 manufacturing industries and 54 years from 1958 to 2011. We dropped the following observations: 156 with missing data on value added, employment, capital, wage rate, and profit rate; 81 with negative and zero real total profits. The remaining 25,305 observations are used in the analysis. This is the same cleaning that firm-level data from censuses are subjected to.

With these data, the distribution of the labor share (shown in figure A1) varies significantly, from close to 0 to almost 1.

Figure A1. Distribution of the Employment Share in Value Added



Source: Authors

Table A1 shows key summary statistics of the variables.

Table A1. Descriptive Statistics of the Dataset

	Mean	**Std. Dev.**	**Min**	**Max**
Wage rate (w) in 1997 USD (annual)	29,802	26,343	129	1,942,914
Profit rate (r) in %	82.17	68.03	0.31	3,734
Employment (L) number of workers	34,075	43,759	200	559,900
Capital (J) in 1997 USD million	2,801	6,484	4.10	133,000
Materials (Z) in 1997 USD million	3,437	9,983	1.33	411,086
Value added (V) in 1997 USD million	2,997	10,924	1.71	672,000
Total output (Y) in 1997 USD million	6,434	19,088	3.04	898,000
Employment share in value added	0.41	0.13	0.02	0.99
Capital share in value added	0.59	0.13	0.01	0.98
Employment share in total output	0.21	0.09	0.01	0.74
Capital share in total output	0.29	0.10	0.01	0.89
Materials share in total output	0.50	0.12	0.09	0.96

Source: Authors

The unlikely values of the profit rates that we obtained do not mean that what we did to construct the identity is incorrect. It means that at least one of the series used to construct the profit rate (e.g., total profits, stock of capital) is probably wrong. Moreover, it ought to be clear that this does not affect our exercise because the identity holds. Nevertheless, appendix 3 discusses an alternative to this profit rate.

The problem is that, while indeed the identity must hold, finding the correct empirical counterparts (i.e., all the variables that constitute the identity) is not simple, even in these databases.

As indicated in email exchanges, there are two major caveats in constructing the identities using the database. First, one might think that the calculated total output as the sum of value added and the total cost of materials (both in nominal terms) should be equal to total output in the database (*vship*). The two numbers are close but not equal since value added (*vadd*) in the database includes adjustments for changes in inventories and some other things. Therefore, we used the identity total output (*vship*) = total wage bill (*pay*) + total profits + total materials cost (*matcost*) to residually calculate total profits, and then compute a new value-added series—all terms are deflated by the price deflator *piship*.

Second, some costs (e.g., purchased services, taxes) that are missing in the database are now being collected in the Annual Survey of Manufactures (ASM). We used the 2018 ASM data to check how much these costs represent in total output. We found out that in 2018 (356 six-digit industries), these costs accounted for only 7.4 percent of the total output: = total other costs/(total output (*vship*) + total other costs).

APPENDIX 2: GROSS OUTPUT REGRESSIONS

The accounting identity (5) for total output can be rewritten as:

$$Y \equiv B w^{\alpha} r^{\beta} L^{\alpha} J^{\beta} Z^{1-\alpha-\beta} \qquad \text{(A1)}$$

where α, β, and $(1-\alpha-\beta)$ are the shares in total output of employment, capital, and materials, respectively.

Cobb-Douglas results for total output are shown in table A2. Very different results were obtained with the five estimators. Although the estimates obtained with LSDV, IV, and GMM add up to about one (constant returns to scale), the coefficients of employment are very low, while those of materials is very high (0.88, 0.93). GMM estimates are very poor, with very low elasticities of employment (0.002 and statistically insignificant) and capital. In the case of L-P, we show two results, one using two moment conditions and the other one with five. The elasticity of materials is statistically insignificant in both cases. Once again, overall, OLS seems to yield the most sensible results.

Table A2. Cobb-Douglas Regressions: Dependent Variable Is the Logarithm of Total Output ($\boldsymbol{lnY}$)

	OLS	**LSDV**	**IV**[33]	**System GMM**[34]	**L-P 2 moments**	**L-P 5 moments**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Constant	2.10 (94.9)	0.55 (17.8)	0.58 (18.3)	1.01 (6.91)		
Gross output lagged (Y_{t-1})				0.24 (18.4)		
Employment (L)	0.14 (84.5)	0.06 (33.1)	0.06 (33.0)	0.002 (0.16)	0.16 (17.2)	0.16 (13.5)
Capital (J)	0.12 (65.3)	0.10 (54.0)	0.10 (51.8)	0.03 (2.27)	0.65 (3.34)	0.82 (5.08)
Materials (Z)	0.75 (386)	0.88 (478)	0.88 (471)	0.93 (56.9)	0.14 (0.55)	0.01 (0.64)
Sector fixed effects		Yes	Yes	Yes		
R^2	0.973	0.993	0.993	N.A.	N.A.	N.A.
Degree of returns to scale	1.011 (826)	1.036 (592)	1.035 (579)	0.971 (70.1)	0.951 (0.04)	0.991 (0.00)
No. of observations	25,305	25,305	24,826	24,826	25,305	25,305

Source: Authors.
Notes: (i) Regressors are expressed in logarithm; (ii) columns (1)–(2): t-values are in parentheses, columns (3)–(6): z-values are in parentheses; (iii) columns (1)–(4): z-values are in parentheses under degrees of returns to scale; (iv) column (4) shows the long-run elasticities of employment, capital, and materials (z-values in parenthesis); (v) columns (5)–(6): chi-squared (Wald test) is in parentheses to test the null that the degree of returns to scale is 1; (vi) In the L-P regressions, Stata's default option to solve the model is nonlinear least squares (NLLS), based on Newton's method. We chose instead the option of a two-dimensional grid search. Candidate values for the elasticities of capital and materials range from 0.01 to 0.99, in increments of 0.01. Although much slower than NLLS, the grid search is handy for confirming that NLLS has found the global minimum of the objective function. Moreover, if there is insufficient variation in the capital and proxy variables, NLLS may have difficulty solving the minimization problem.

Estimates of the accounting identity for total output are shown in table A3. For reference, the average employment, capital, and materials shares in total output are 0.21, 0.29, and 0.59, respectively (across the entire dataset). Estimates are not far from the respective factor shares,

[33] A variety of potential instruments were tested, but in most cases results were poor. Results reported use capital lagged one period as the instrumental variable. Estimation is two-stage least squares.
[34] Estimation as in table 1. Tests for autocorrelation and validity of the instruments: the z-values of AR(1) = -12.51 and AR(2) = -8.05. This means that the null hypothesis that there is no second-order serial correlation is rejected. Chi-squared Sargan test = 13,134 and the chi-squared Hansen test = 443. This means that the null hypothesis that the instruments are exogenous is rejected.

with the exceptions of the estimates of materials and the capital stock in columns (6) and (7) (in one case very low and in the other one very high).

Table A3. The Accounting Identity: Dependent Variable Is Total Output (lnY)

	OLS	LSDV (sector dummies)	LSDV (wage-profit-dummies)	IV[35]	System GMM[36]	L-P 2 moments	L-P 7 moments
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	1.03 (91.3)	1.14 (76.9)	1.25 (13.6)	1.15 (76.8)	1.25 (11.5)		
Gross output lagged (Y_{t-1})					0.05 (8.01)		
Wage rate (w)	0.16 (113)	0.19 (111)		0.19 (111)	0.20 (13.9)	0.17 (12.2)	0.17 (12.3)
Profit rate (r)	0.28 (266)	0.27 (256)		0.27 (260)	0.22 (20.4)	0.29 (21.2)	0.29 (22.1)
Employment (L)	0.16 (187)	0.19 (147)	0.12 (17.2)	0.19 (147)	0.17 (16.9)	0.16 (18.2)	0.16 (16.8)
Capital (J)	0.30 (279)	0.27 (260)	0.34 (37.7)	0.28 (260)	0.23 (22.5)	0.55 (4.88)	0.08 (2.02)
Materials (Z)	0.54 (602)	0.53 (386)	0.54 (65.4)	0.53 (387)	0.57 (40.4)	0.28 (2.12)	0.78 (17.3)
Sector fixed effects		Yes	Wage/profit dummies	Yes	Yes		
R^2	0.996	0.999	0.999	0.999	N.A.	N.A.	N.A.
Degree of returns to scale	0.999 (2,161)	0.997 (1,308)	1.004 (254)	0.997 (1,298)	0.965 (170)	0.993 (50.9)	1.023 (76.0)
No. of observations	25,305	25,305	473 Year: 2011	24,826	24,826	25,305	25,305

Source: Authors.

Notes: (i) Regressors are expressed in logarithm; (ii) columns (1)–(3): t-values are in parentheses, columns (4)–(7): z-values are in parentheses; (iii) columns (1)–(7): z-values are in parentheses under degrees of returns to scale; (iv) column (5) shows the long-run elasticities of employment, capital, and materials (z-values in parenthesis); (v) In the L-P regressions, Stata's default option to solve the model is NLLS, which is based on Newton's method. We chose instead the option of a two-dimensional grid search. Candidate values for the elasticities of capital and materials range from 0.01 to 0.99, in increments of 0.01. Although much slower than NLLS, the grid search is handy for confirming that NLLS has found the global minimum of the objective function. Moreover, if there is insufficient variation in the capital and proxy variables, NNLS may have difficulty solving the minimization problem; (vi) column (7): estimation is based on 7 moment conditions, the same 5 moment conditions used to estimate the regression in column (6) of table A2 plus the corresponding moment conditions for the wage rate and the profit rate.

[35] Results reported use capital lagged one period as the instrumental variable. Estimation is two-stage least squares. Since what is being estimated is an accounting identity, tests for the validity of the instrument do not apply.

[36] Estimation as in table 1. Since what is being estimated is an accounting identity (or an equation very close to it), the tests for autocorrelation and validity of the instruments do not apply.

APPENDIX 3: ESTIMATION RESULTS WITH THE ALTERNATIVE DATASET

An alternative to the profit rate implicit in the distribution accounting identity is to artificially generate a rental cost of capital (ω), assumed in the literature to be the marginal product of capital. This gives a different distribution of the factor shares. For robustness, we have also estimated the regressions with the alternative dataset. This appendix shows that our results are robust to this alternative.

We constructed hypothetical rental rates (user cost) of capital as follows:

(i) We generated 25,542 uniformly distributed random numbers, for each of the following ten ranges: (0.05,0.25), (0.04,0.07), (0.05,0.10), (0.06,0.12), (0.03,0.05), (0.03,0.04), (0.07,0.13), (0.043,0.123), (0.04,0.083), (0.08,0.09). For each of the ten ranges we had 473 groups (industries) and 54 observations for each (years), i.e., 25,542 observations in each range. We arranged the ten series in ascending order;

(ii) We repeated step (i) but now arranged the ten series in descending order.

(iii) We repeated step (i) but did not rearrange the randomly generated numbers. This gives a total of 30 series of (random) numbers, each with 25,542 observations (473 industries times 54 years), for a total of 766,260 observations.

(iv) We assigned increasing profit rates to 160 industry codes, decreasing profit rates to 157 industry codes, and uniform distribution rates to the remaining 156 industry codes, each 54 observations (years).

While theoretically the rental rate is the price per unit of capital (e.g., dollars per square meter of office), in practice it is also a percentage. We recalculated total profits (P') as the product of this alternative rate (ω) times the total real capital stock (J). This allows us to create (hypothetical) total costs (TC) as:

$$TC \equiv wL + \omega J \qquad \text{(A2)}$$

which is an accounting identity, too. TC is also a value measure. Note that (A2) is consistent with the value-added accounting identity (4) since the latter can be written as:

$$V \equiv W + P' + \Pi \equiv wL + \omega J + \Pi \qquad \text{(A3)}$$

where Π denotes extra or excess profits. Nothing changes conceptually as identities (4) ($V \equiv wL + rJ$) and (A11) are the same because $rJ \equiv \omega J + \Pi$. This was Samuelson's (1979, 932) remark that $P = rJ$ is the residually computed vector of profits ($V - wL$). It may or may not be true that excess profits (Π) can be written as the product of another profit rate (e.g., ω') times the capital stock (J). This does not undermine the argument that the identities hold. This means that the precise values and path of ω (and of r) are immaterial. All that matters is that value added, total costs, employment, wage rate, rental price, capital stock, and excess profits are all variables definitionally related through the accounting identity.

Expression (A2) gives rise to the *dual* of total factor productivity growth used sometimes, and calculated as $\widehat{TFP}^D = a^c\widehat{w} + (1 - a^c)\widehat{\omega}$, where $a^c = (wL)/TC$ is the employment share in total costs and $(1 - a^c) = (\omega J)/TC$ is the capital share. In general, the dual and that above derived from accounting identity (12), i.e., $\widehat{TFP} = a\widehat{w} + (1 - a)\hat{r}$, will differ. Note that the growth decomposition can also be based on (A3) with the shares in value added, parallel to equation (12), but now including the growth rates of ω and Π. The important point is that if what is being estimated is $V = f(L, J)$, the bias in the estimation of the elasticities will be due to the omission of the wage and profit rate, however the latter is split.

Theoretically, the dual of TFP growth is derived and estimated from the cost function $TC = f(w, \omega, Q, \lambda)$, where λ denotes technical progress, and it is interpreted as the rate of cost reduction. In empirical applications, however, the measure of output is not Q but value added, V. Given a precise functional form (e.g., Cobb-Douglas cost function), the rate of technical change is obtained by solving it for the growth in unit costs (i.e., the growth of TC/V). The rate of technical change equals $\lambda = a^c\widehat{w} + (1 - a^c)\widehat{\omega}$, the same expression as that given by the accounting identity (A2).

As before, equation (A3) can be differentiated with respect to time and then integrated under the assumption that factor shares are constant. This yields:

$$V_t \equiv B w_t^{\bar{a}} \omega_t^{\bar{b}} L_t^{\bar{a}} J_t^{\bar{b}} \Pi_t^{1-\bar{a}-\bar{b}} \quad \text{(A4)}$$

where a bar on top of the variable denotes the corresponding average share in value added.

Table A4 provides descriptive statistics of the dataset based on the alternative profit rate.

Table A4. Descriptive Statistics of the Alternative Dataset

	Mean	**Std. Dev.**	**Min**	**Max**
Wage rate (w) in 1997 USD (annual)	30,003	26,666	479	1,942,914
Profit rate (ω) in %	7.64	3.78	3.00	25.00
Employment (L) number of workers	33,762	43,077	200	559,900
Capital (J) in 1997 USD million	2,730	6,107	4.10	133,347
Materials (Z) in 1997 USD million	3,392	9,826	9.51	411,086
Total cost in 1997 USD million	1,250	2,484	9.12	99,067
Extra profits (Π) in 1997 USD million	1,737	8,944	0.13	591,337
Value added (V) in 1997 USD million	2,986	10,907	14.31	671,663
Total output (Y) in 1997 USD million	6,378	18,964	23.82	898,019
Employment share in total cost	0.84	0.12	0.16	0.99
Capital share in total cost	0.16	0.12	0.006	0.84
Employment share in value added	0.41	0.13	0.02	0.96
Capital share in value added	0.08	0.07	0.0008	0.77
Excess profits share in value added	0.51	0.14	0.0005	0.97

Source: Authors
Note: The dataset contains 25,186 observations.

Table A5 reports estimates of the "production function" consistent with equation (A4). It contains the variable excess profits as a regressor, but not the factor rewards, that is:

$$ln\, V_{it} = c + \gamma_3\, ln\, L_{it} + \gamma_4\, ln\, J_{it} + \gamma_5 ln\Pi_{it} + \varepsilon_{it} \quad \text{(A5)}$$

Certainly, equation (A5) is not a standard production function because it adds the variable extra profits. The objective of this exercise is to highlight that this regression is the same as those shown in table 1 but now adding this variable. Here, the accounting identity has been reconstructed as a result of using an alternative profit rate, the consequence of which has been to add the variable excess profits, which, naturally, is highly significant.

Table A5. Cobb-Douglas Regressions with the Alternative Profit Rate: Dependent Variable Is the Logarithm of Value Added (lnV)

	OLS	LSDV	IV[37]	GMM[38]	L-P
	(1)	(2)	(3)	(4)	(5)
Constant	3.05 (162)	3.10 (80.0)	3.14 (82.9)	2.81 (16.2)	
Value added lagged (V_{t-1})				0.42 (14.5)	
Employment (L)	0.26 (176)	0.24 (123)	0.23 (124)	0.18 (11.6)	0.25 (28.7)
Capital (J)	0.17 (116)	0.20 (89.5)	0.19 (85.8)	0.09 (5.56)	0.19 (2.33)
Extra profits (Π)	0.58 (380)	0.56 (329)	0.57 (339)	0.62 (39.0)	0.57 (33.6)
Sector fixed effects		Yes	Yes	Yes	
R^2	0.978	0.989	0.990	N.A.	N.A.
Degree of returns to scale	1.014 (954)	0.999 (460)	0.994 (469)	0.887 (61.6)	1.015 (0.03)
No. of observations	25,345	25,345	24,835	24,835	25,345

Source: Authors
Notes: (i) Regressors are expressed in logarithm; (ii) columns (1)–(2): t-values are in parentheses, columns (3)–(5): z-values are in parentheses; (iii) columns (1)–(4): z-values are in parentheses under degrees of returns to scale; column (5): chi-squared (Wald test) is in parentheses to test the null that the degree of returns to scale is 1.

Finally, table A6 reports the regressions of the complete value-added accounting identity (A4), parallel to the regressions in table 2. The OLS regression is:

$$ln\, V_{it} = c + \gamma_1 lnw_{it} + \gamma_2 ln\omega_{it} + \gamma_3\, ln\, L_{it} + \gamma_4\, ln\, J_{it} + \gamma_5 ln\Pi_{it} + \varepsilon_{it} \qquad \text{(A6)}$$

[37] A variety of potential instruments were tested but in most cases results were poor. Results reported use capital lagged one period as the instrumental variable. Estimation is two-stage least squares.
[38] Estimation as in table 1. Tests for autocorrelation and validity of the instruments: the z-values of AR(1) = -10.03 and AR(2) = -5.03. This means that we reject the null hypothesis that there is no second-order serial correlation. The chi-squared Sargan test = 8,434 and the chi-squared Hansen test = 452. This means that we reject the null hypothesis that the instruments are exogenous.

and as before, coefficients are estimated unrestricted. Naturally, it is expected that the estimated coefficients be close to the average shares of employment, capital, and extra profits in value added, that is, $\gamma_1 \simeq \gamma_3 \simeq \bar{a}$=0.41, $\gamma_2 \simeq \gamma_4 \simeq \bar{b}$=0.08 and $\gamma_5 \simeq (1 - \bar{a} - \bar{b})$=0.51, respectively (see table A2). As before, when we compared the results in table 1 and table 3, now the results in table A4 show elasticities closer to the factor shares than those in table A3. This reconfirms that the problem (i.e., bias) estimating putative production functions does not derive from the regressors' endogeneity but from omitting (or incorrectly approximating) the factor rewards.

Also, estimation of LSDV in column (3) for 2011 uses 200 wage-profit rate dummies. The average shares of employment, capital, and residual profits in 2011 are $\bar{a}$=0.31, $\bar{b}$=0.10, and $1 - \bar{a} - \bar{b}$=0.59, respectively. Once again, these results indicate that any dataset consistent with the distribution accounting identity (however this is written) has to produce estimates of the elasticities close to the factor shares.

Table A6. The Accounting Identity with the Alternative Profit Rate: Dependent Variable Is Value Added (lnV)

	OLS	LSDV (sector dummies)	LSDV (wage-profit dummies)	IV[39]	GMM[40]	L-P
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	0.90 (46.7)	0.79 (25.7)	3.12 (19.7)	0.82 (27.5)	0.35 (1.46)	
Value added lagged (V_{t-1})					0.20 (11.1)	
Wage rate (w)	0.40 (167)	0.50 (165)		0.49 (169)	0.52 (23.1)	0.41 (23.4)
Profit rate (ω)	0.08 (54.5)	0.09 (36.5)		0.09 (38.6)	0.08 (2.72)	0.09 (13.6)
Employment (L)	0.39 (305)	0.46 (242)	0.26 (24.1)	0.45 (248)	0.49 (25.3)	0.39 (28.0)
Capital (J)	0.12 (118)	0.13 (82.6)	0.15 (13.1)	0.12 (78.4)	0.08 (7.94)	0.22 (4.33)
Extra profits (Π)	0.48 (400)	0.41 (277)	0.59 (57.7)	0.43 (300)	0.38 (21.0)	0.48 (24.2)
Sector fixed effects		Yes	Wage/profit dummies	Yes	Yes	
R^2	0.990	0.995	0.996	0.995	N.A.	N.A.
Degree of returns to scale	0.998 (1,404)	0.999 (674)	0.999 (161)	0.996 (700)	0.949 (84.7)	1.084 (23.3)
No. of observations	25,345	25,345	470 Year: 2011	24,835	24,835	25,345

Source: Authors
Notes: (i) Regressors are expressed in logarithm; (ii) columns (1)–(3): t-values are in parentheses, columns (4)–(6): z-values are in parentheses; (iii) columns (1)–(6): z-values are in parentheses under degrees of returns to scale; (iv) column (5) shows the long-run elasticities of employment, capital, and extra profits (z-values in parenthesis).

[39] Results reported use capital lagged one period as the instrumental variable. Estimation is two-stage least squares. Since what is being estimated is an accounting identity, tests for the validity of the instrument do not apply.
[40] Estimation as in table 1. Since what is being estimated is an accounting identity (or something very close to it), the tests for autocorrelation and validity of the instruments do not apply.

APPENDIX 4: KALMAN FILTER ESTIMATION OF THE PRODUCTION FUNCTION WITH REFERENCE TO THE CES PRODUCTION FUNCTION

This appendix provides time-varying estimates of a version of equation (12) in state-space form. Equation (A7) is a signal equation that captures the accounting identity in equation (12):

$$\hat{V}_t = c_t + \sigma_{1t}\hat{w}_t + \sigma_{2t}\hat{r}_t + \sigma_{3t}\hat{L}_t + \sigma_{4t}\hat{J}_t + v_t \quad \text{with } \; v_t \sim iid\; N(0, \sigma_v^2) \qquad \text{(A7)}$$

where σ_{1t}, σ_{2t}, σ_{3t}, and σ_{4t} are the coefficients of the growth rates of the wage rate, profit rate, employment, and real capital stock, respectively, at time t. Following the standard procedure in the literature on state-space modeling (Harvey 1989), to capture possible level breaks or trend patterns the state equations are modeled as unit roots, e.g., $\sigma_{1t} = \sigma_{1t-1} + w_t$, with $w_t \sim iid\; N(0, \sigma_w^2)$, and where σ_{1t} is the unobservable state vector, while w_t is an i.i.d. noise component. The five coefficients are estimated unrestricted. The model is estimated via the Kalman smoothing procedure. This procedure differs from the Kalman filter in the construction of the state series, as the latter technique uses only the information available up to the beginning of the estimation period. Smoothed series tend to produce more gradual changes than filtered ones and, as discussed by Sims (2001), they provide more precise estimates of the actual time variation in the data.[41]

We show as an example the results corresponding to one time-series data, 54 years from 1958 to 2011, sector 333220 (Plastics and Rubber Industry Machinery Manufacturing). Figure A2 shows that the coefficients of the growth rate of the wage rate and employment growth are approximately equal to the actual labor share. Figure A3 shows that the coefficients of the growth rate of the profit rate and of the growth rate of the capital stock are also approximately the same as the actual capital share. The sum of the coefficients of the growth rate of the wage rate and profit rate ranges from 0.94 to 1.14, while the sum of the coefficients of employment growth and capital stock growth ranges from 0.94 to 1.07. Clearly, it is the identity that drives these results.

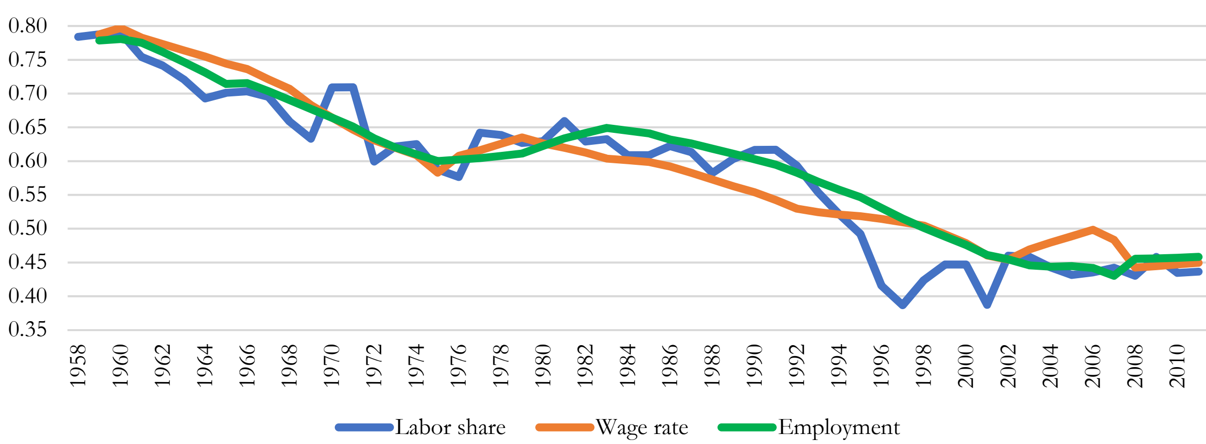
[41] Kim (2006) showed that conventional Kalman estimation of a time-varying parameter model leads to invalid inferences in the presence of endogenous regressors. Since we have argued throughout the paper that this is not our case, we do not implement the two-step instrumental variable procedure that he proposed.

The exercise is revealing in that the factor shares are trending, hence the logic would be not to estimate a Cobb-Douglas form. Indeed, one could argue that what lies behind the trending factor shares is a CES production function, which in growth rates is:

$$\hat{V}_t = \lambda + \left[\frac{\delta L_t^{-\rho}}{D_t}\right]\hat{L}_t + [\frac{(1-\delta)J_t^{-\rho}}{D_t}]\hat{J}_t \qquad \text{(A8)}$$

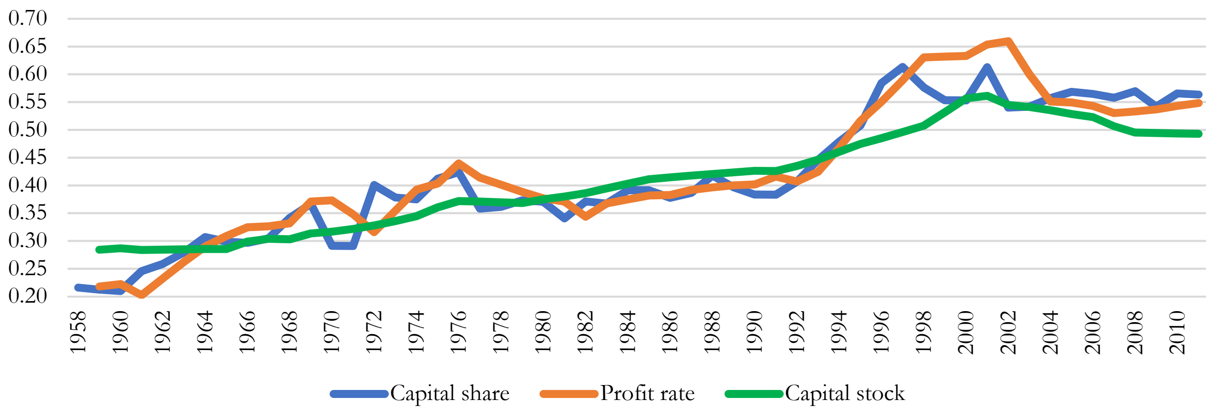
where λ is the exogenous rate of technical progress and $D_t = \delta L_t^{-\rho} + (1-\delta)J_t^{-\rho}$.

Figure A2. Actual Labor Share and Estimated Coefficients of Growth Rates of the Wage Rate and Employment



Source: Authors' estimates

Figure A3. Actual Capital Share and Estimated Coefficients of the Growth Rates of the Profit Rate and the Capital Stock



Source: Authors' estimates

Our argument is that, looking at identities (12) and (A7), it is self-evident that equation (A8) will work only if the expressions $\frac{\delta L_t^{-\rho}}{D_t}$ and $\frac{(1-\delta)J_t^{-\rho}}{D_t}$ proxy well the labor and capital shares, a_t and $(1 - a_t)$, respectively, and the constant λ proxies well the weighted average $a_t\hat{w}_t + (1 - a_t)\hat{r}_t$. All this would show (assuming it works) is that equation (A8) tracks well equation (A7), the identity, and not that equation (A8) is the true production function. The reason for the increasing labor share could be, for example, a decline in labor union power, in which case $a_t/(1 - a_t)$ would decline over time, which has nothing to do with a CES production function.