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Does the aggregate production function imply anything about the laws of production? A comment

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This paper discusses McCombie's (1987) empirical evidence on the Simon–Shaikh critique. It is argued that while the theoretical argument is correct, the evidence presented depends on the adjustment of the stock of capital for capacity utilization. Probably factor shares are not sufficiently constant in this data set for the Cobb–Douglas form to provide a good approximation to the accounting identity.

I. INTRODUCTION

McCombie (1987) illustrated (with an example) that the good fit obtained by estimating the aggregate production function can yield no insights into the underlying structure of production. Following the work of Shaikh (1974, 1980) and Simon (1979), he showed that aggregate production functions are nothing but approximations to the total cost accounting identity. The purpose of this note is to point out some problems in McCombie's empirical analysis for the Australian manufacturing sector. These problems, nevertheless, do not undermine the essence of the Simon and Shaikh critiques.

To begin with, consider the value of total output (i.e., national income) at time t, defined as:

$$TC_t = Q_t = w_t L_t + r_t K_t \tag{1a}$$

where TC, Q, w, r, L, and K denote total costs, output, wage rate, profit rate, level of employment and stock of capital, respectively. Expression 1a in growth rates is as follows:

$$\dot{q}_t = a_t \dot{w}_t + (1 - a_t)\dot{r}_t + a_t \dot{l}_t + (1 - a_t)\dot{k}_t \tag{1b}$$

where a_t denotes the labour share in total output, and the dots denote growth rates. If we assume constant factor shares, i.e., $a_t = a$, substitute in Equation 1b, and integrate we obtain

$$Q_t = A w_t^a r_t^{1-a} L_t^a K_t^{1-a} \tag{2a}$$

If we further assume that wages and the profit rate grow at the constant rates ϕ_w and ϕ_r, respectively, that is, $w_t = w_0 \exp(\phi_w t)$ and $r_t = r_0 \exp(\phi_r t)$, then Equation 2a can be rewritten as

$$Q_t = B e^{[a\phi_w + (1-a)\phi_r]t} L_t^a K_t^{1-a} \tag{2b}$$

where A and B are constants. Note that Expression 2b is identical with the Cobb–Douglas form with constant returns to scale, but directly derived from the accounting identity, without any reference to production functions. The econometric implication is that if the previous two assumptions about factor shares, wages, and profit rate happen to be true, and we estimate this production function expressed in logarithms, that is

$$\ln Q_t = c + \phi t + \alpha \ln L_t + \beta \ln K_t + v_t \tag{3}$$

where v_t is the error term, α and β are the parameter elasticities, and ϕ is interpreted as the constant growth rate of total factor productivity, we should expect a perfect fit, with the estimates of α and β being equal to the constant shares of labour and capital, i.e., a and $(1 - a)$, and ϕ equal to the weighted average of the constant growth rate of wages and profit rate (i.e., $\phi = a\phi_w + (1 - a)\phi_r$). Simply put, this is an identification-interpretation problem (McCombie, 1987; McCombie and Dixon, 1991). Since the initial expression for the total cost, i.e., Expression 1a, is compatible with any production function, one cannot draw any inference about the aggregate elasticity of substitution. The conclusion is that if what we are estimating is an (approximation to) identity, the whole exercise is pointless, and high coefficients of determination should be anticipated.

II. EMPIRICAL EVIDENCE

In this section we review the empirical evidence presented by McCombie (1987) in order to illustrate the Simon–Shaikh critique. To this purpose, we use the data set for the Australian manufacturing industry compiled by Burke and Naughtin (1984) (B–N) covering the period 1950–81, and containing output, employment, capital stock, and factor shares. This is the same data set used by McCombie (1987), except for the fact that McCombie adjusted the stock of capital for capacity utilization. However, B–N does not provide the utilization rate, and neither does McCombie. For reference, the regressions are (Table 1 in McCombie's paper):

$$\ln Q = a_1 + b_1 \ln w + b_2 \ln r + b_3 \ln L + b_4 \ln K + \psi_1 \quad \text{(a)}$$

$$\ln Q = a_2 + b_5 \ln L + b_6 \ln K + \psi_2 \quad \text{(b)}$$

$$\ln Q = a_3 + c_1 t + b_7 \ln L + b_8 \ln K + \psi_3 \quad \text{(c)}$$

$$\ln(Q/L) = a_4 + c_2 t + b_9 \ln(K/L) + \psi_4 \quad \text{(d)}$$

$$\ln w = a_5 + b_{10} \ln L + b_{11} \ln K + \psi_5 \quad \text{(e)}$$

$$\ln r = a_6 + b_{12} \ln L + b_{13} \ln K + \psi_6 \quad \text{(f)}$$

$$\ln w = a_7 + c_3 t + \psi_7 \quad \text{(g)}$$

$$\ln r = a_8 + c_4 t + \psi_8 \quad \text{(h)}$$

The results of our estimations are shown in Table 1.[1] The following conclusions can be drawn:

1. The results are slightly different from those of McCombie (ours worse in general). Two reasons account for this. First, the estimation period is different. Our regressions are estimated for 1950–81 or 1951–81, while McCombie's for 1953–81. Second, and probably more important, McCombie adjusted the capital stock series for capacity utilization. What is interesting, however, is that Equation (a) yields the same parameters, but with different t-statistics and Durbin–Watson. Also Regressions (g) and (h), which do not involve the stock of capital, yield different results.
2. McCombie's Equation (a) gives a perfect fit with an R^2 of 1. He concluded that '... factor shares are *sufficiently stable* for the estimation of the identity equation to give a good fit with the R^2 approximately equal to unity' (McCombie, 1987, p. 1128) (italics added). Regression (a) must certainly yield a very high R^2 as long as factor shares are sufficiently constant. However, paradoxically, one will never be able to obtain an R^2 of 1, as McCombie's Table 1 reports.[2] The reason is that an R^2 of 1 can be achieved if and only if factor shares are entirely constant for the estimation period (i.e., we doubt if there has ever been such case in actual economies). But should this occur, perfect multicollinearity, derived from the fact that the series are linked through the cost identity (1a), would prevent OLS from producing any estimate (Felipe, 1997). But the question is whether it is true that factor shares were sufficiently constant in the case at hand for the Cobb–Douglas to provide a correct approximation to the identity. This is an empirical question.[3] We notice (see Figure 1) that the labour share oscillated between a maximum of 0.758 and a minimum of 0.639 (i.e., twelve percentage points of difference in a period of thirty years), with an average of 0.686 and a standard deviation of 0.037. Can the Cobb–Douglas provide a good fit to these data? To gain insight into this question, we estimate the rolling coefficients for Regression (a) with a window of 20 periods. Figures 2–5 plot the results. As can be seen, the estimates are not stable, and clearly display a structural break in the early 1970s.[4] Then, there are two possibilities: one is to fit other forms that accommodate the fact that factor shares are not constant, such as the translog. Following this route we would remain within the confines of the Simon–Shaikh critique.[5] The second solution, less illuminating for the purposes at hand, is to keep the Cobb–Douglas form and try to adjust the original series, for example the capital stock, for capacity utilization.
3. McCombie showed that $\ln w_t$ and $\ln r_t$, are well proxied by a linear time trend. This was simply done by regressing the variables on a linear trend (i.e., Equations (g) and (h)). These regressions, based on the assumption that wages and profit rate are trend stationary processes, are incorrect *a priori*. As we know, it is difficult to distinguish a trend stationary process from a random walk with drift. If wages and the rental price of capital are integrated processes, a time trend on the right-hand side will have the effect of a spurious detrending. Under these circumstances, R^2, t-statistics, and Durbin–Watson are functionals of Wiener processes (Phillips, 1986). A significant time trend in a regression with $I(1)$ series is almost certainly a reflection of the spurious regression phenomenon which arises when

[1] McCombie fitted the regressions for the period 1953–81; however, Burke and Naughtin's data set is for the period 1950–81 (1951–81 for the factor shares). We used the complete period.
[2] It is understood that McCombie wrote $R^2 = 1$ when he obtained $R^2 = 0.999$.
[3] See Felipe and Holz (1996) for a simulation analysis.
[4] Results change slightly with other window sizes.
[5] The factor shares corresponding to the translog production function take the form $a_t = \alpha_L + B_{KL} \ln K_t + B_{Lt} t + B_{LL} \ln L_t$ (i.e., this is the condition to derive the translog from the accounting identity). This approximation provides a better representation of a_t, than the constant a. However, due to the presence of multicollinearity, a translog does not fit the data well.

Table 1. *Australian manufacturing sector: 1950–81*

Equation	t	$\ln w$	$\ln r$	$\ln L$	$\ln K$	$\ln(K/L)$
(a)		0.75 (38.63)	0.317 (61.13)	0.682 (62.89)	0.276 (20.52)	
			$R^2 = 0.999$; D.W. = 1.32; $\chi_1^2 = 13.12$; $\chi_2^2 = 5.61$, ADF(2) = −2.69			
(b)				0.95 (13.39)	0.63 (36.53)	
			$R^2 = 0.997$; D.W. = 0.72; $\chi_1^2 = 7.40$; $\chi_2^2 = 0.21$; ADF(2) = −1.82			
(c)	0.052 (3.09)			1.57 (7.48)	−0.41 (−1.21)	
			$R^2 = 0.998$; D.W. = 0.61; $\chi_1^2 = 12.82$; $\chi_2^2 = 1.55$; ADF(2) = −2.09			
(d)	0.07 (12.27)					−0.76 (−6.00)
			$R^2 = 0.996$; D.W. = 0.69; $\chi_1^2 = 12.18$; $\chi_2^2 = 1.59$; ADF(2) = −2.04			
(e)				−0.49 (−6.33)	0.79 (42.15)	
			$R^2 = 0.995$; D.W. = 1.11; $\chi_1^2 = 10.23$; $\chi_2^2 = 2.38$, ADF(2) = −2.69			
f)				2.02 (6.99)	−0.73 (−10.48)	
			$R^2 = 0.82$; D.W. = 0.44; $\chi_1^2 = 0.20$; $\chi_2^2 = 1.02$; ADF(2) = −1.65			
(g)	0.039 (87.19)					
			$R^2 = 0.996$; D.W. = 0.81; $\chi_1^2 = 4.09$; $\chi_2^2 = 1.28$; ADF(2) = −2.62			
(h)	−0.018 (−6.32)					
			$R^2 = 0.58$; D.W. = 0.24; $\chi_1^2 = 21.45$; $\chi_2^2 = 2.22$; ADF(2) = −1.70			

Note: t-statistics in parentheses. χ_1^2 is Ramsey's RESET test for functional form using the square of the fitted values. χ_2^2 is the Bera-Jarque test for Normality. Critical values for χ_1^2 are 3.84 and 2.71 for 95% and 90% confidence, respectively. Critical values for χ_2^2 are 5.99 and 4.61 for 95% and 90% confidence, respectively; ADF (2) is the ADF test for cointegration with two lags in the residuals. The 95% critical values for these regressions, given the sample size and number of regressors, lie between −4 and −5.
Regressions (a), (e), (f), (g) and (h) are estimated for the period 1951–81.

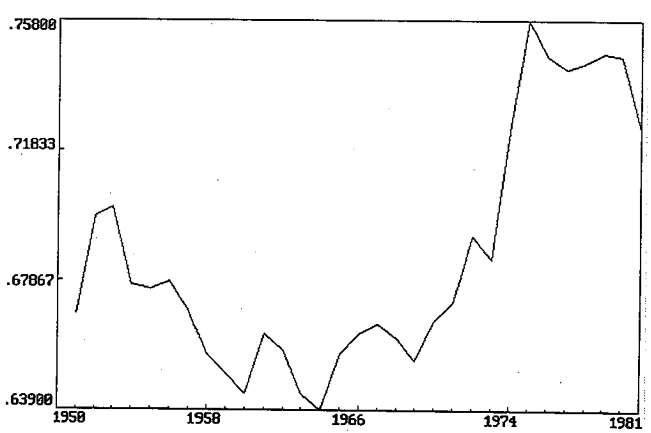

Fig. 1. *Australia's labour share. Manufacturing sector*

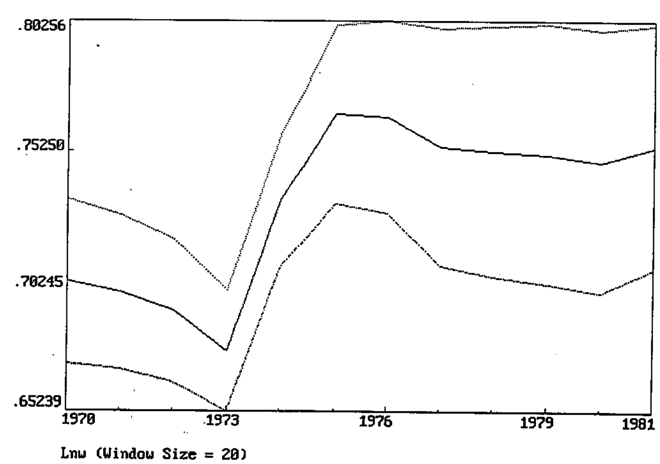

Fig. 2. *Rolling coefficient and standard error*

time is erroneously included as an explanatory variable in a model containing a unit root. The coefficients of the time trends in Regressions (g) and (h) are the constant growth rates of wages (i.e., $c_3 = \phi_w$) and of the rental price of capital (i.e., $c_4 = \phi_r$). McCombie obtained coefficients of 0.041 and −0.049, respectively. These were highly significant. What appears to be a paradoxical result is that when McCombie fitted Equation (c), he obtained an insignificant coefficient for the time trend. Recall that the time trend in this equation is $\phi = a\phi_w + (1-a)\phi_r$. A

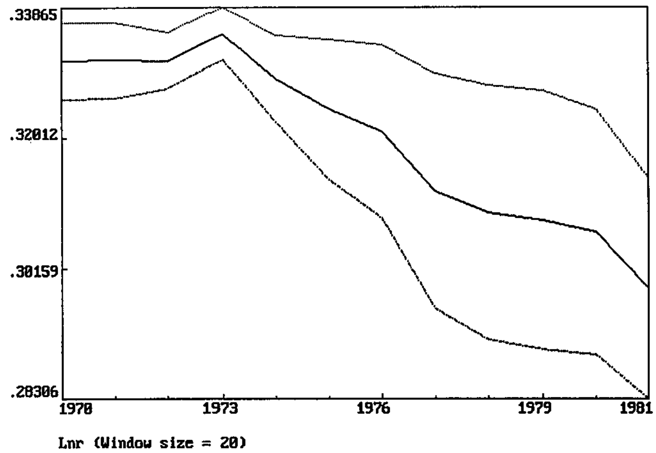

Fig. 3. *Rolling coefficient and standard error*

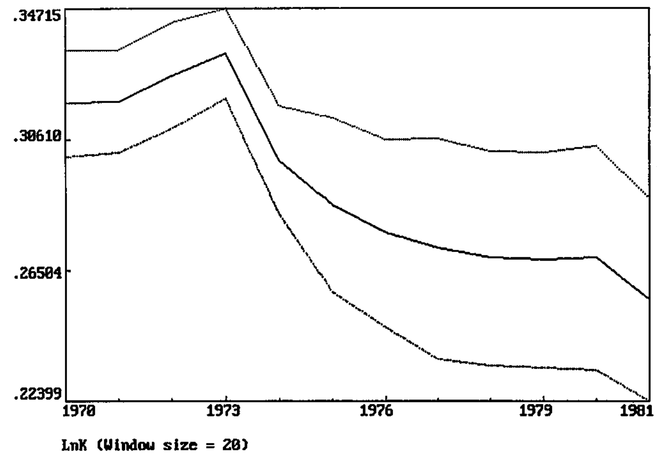

Fig. 5. *Rolling coefficient and standard error*

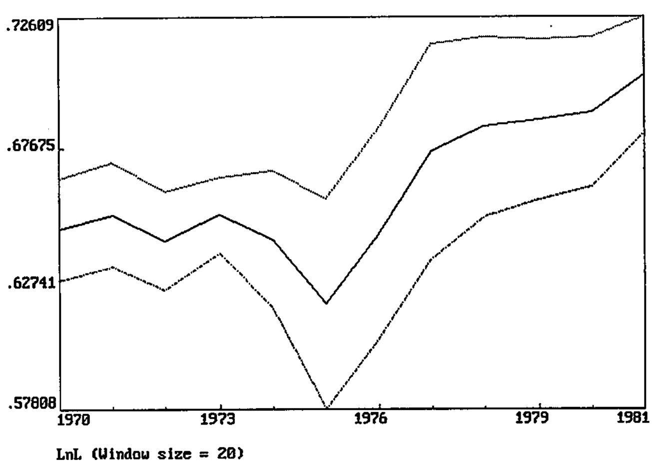

Fig. 4. *Rolling coefficient and standard error*

Table 2. *Augmented Dickey–Fuller test (ADF) for the order of integration*

$$\Delta y_t = \mu + \beta t + \phi y_{t-1} + \sum_{i=1}^{p} \theta_i \Delta y_{t-i} + \varepsilon_t \quad (1)$$

$$\Delta^2 y_t = \mu + \beta t + \phi \Delta y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta^2 y_{t-i} + \varepsilon_t \quad (2)$$

Output	$I(1)$
Employment	$I(1)$
Wages	$I(1)$
Rental price of capital	$I(1)$
Stock of capital[a]	$I(2)$
Labour share[b]	$I(1)$

Notes: Equation 1: ADF test for one unit root: Equation 2: ADF test for two unit roots (Dickey and Pantula, 1987).

[a] The stock of capital is the sum of investment. Because the latter is presumably an integrated series, we started testing for two unit roots in the stock of capital. This hypothesis could not be rejected. Also, recall that this is the original unadjusted series, different from the one McCombie (1987) used.

[b] The series display a marked structural break in 1973. This is confirmed by the recursive least squares estimation of the ADF regressions. The recursive estimates of ϕ indicate marked instability through 1973. However, Perron's (1989) test for the existence of a unit root conditional on the (possible) presence of structural break, does not reject the null hypothesis of a unit root.

coefficient of zero can only be true if $(a\phi_w = -(1-a)\phi_r)$, or if $\phi_w = \phi_r = 0$ (i.e., if one multiplies the estimates of Equations (g) and (h) by the average factor share they do not add up to zero). Another possibility is that $\ln w_t$, and $\ln r_t$ are random walks, in which case their growth rates have an average value of zero. If they are truly random walks they cannot be approximated by any trend stationary function. Table 2 displays the results of the test for unit roots. The Augmented Dickey–Fuller (ADF) test for the residuals seems to indicate that output, labour, wages, rental price of capital, and stock of capital are integrated processes (i.e., the first four of order 1, while the latter of order 2). Certainly, given the small sample size (i.e., 30 years) we must exercise caution in drawing definite conclusions out of these tests. The important aspect, however, is the consideration of the unit root possibility, instead of directly assuming that the underlying model is trend stationary.[6]

[6] The possible spuriousness of the regressions, the order of integration of the variables, as well as their cointegrating properties do not undermine the Simon–Shaikh critique (Felipe and Holz, 1997).

III. CONCLUSIONS

In this note we have re-evaluated McCombie's (1987) empirical evidence of the Simon-Shaikh critique of aggregate production functions. The rationale of the criticism is that to each data set consistent with the value added accounting identity there corresponds a particular production function. We conclude that although the essence of the critique remains valid, McCombie's evidence depends strongly on an aspect outside it, namely, the adjustment of the capital stock for capacity utilization.

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