

## The Tyranny of the Identity: Growth Accounting Revisited

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**ABSTRACT** *It has been argued in the literature that growth accounting may be undertaken by directly differentiating the national income and product accounts identity where total income equals labour's total compensation and total profits. This paper shows that this is simply an exercise in the manipulation of an accounting identity without necessarily having any theoretical foundation. Simulations show that the estimates of total factor productivity growth resulting from growth accounting performed with aggregate monetary data are not equivalent to the true rate of technological progress implied by the micro-data. This suggests that results from the orthodox growth accounting approach may be very misleading.*

**KEY WORDS:** Total factor productivity growth, accounting identity.

**JEL CLASSIFICATION:** O11, O16, O47, O53

### Introduction

A number of papers (Barro, 1999; Hsieh, 1999, 2002) have argued that the neoclassical growth accounting exercises, specifically the derivation of the dual measure of total factor productivity growth (TFPG), can be performed by simply differentiating the National Income and Product Accounts (NIPA) identity. The identity is  $V_t \equiv W_t + \Pi_t$ ,  $\equiv w_t L_t + r_t K_t$  where  $V$  is value added,  $W$  is the total wage bill,  $\Pi$  denotes total profits,  $w$  is the average wage,  $L$  is employment,  $r$  is the rate of profit and  $J$  is the value of the capital stock. The monetary, or value, variables are all measured in real terms.<sup>1</sup>

This approach to growth accounting stands in marked contrast to the arguments of Felipe & McCombie (2003). There it was argued that precisely because of the existence of the accounting identity, growth accounting exercises amount to no more than manipulations of the *ex-post* national income accounting identity, and, as such, they are tautologies without necessarily any behavioural content. The reason for the problem that Felipe & McCombie (2003) highlight is that there is an important difference between using physical data, only available

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at a disaggregated level and which should theoretically be used in growth accounting and production function studies, and data expressed in constant-price monetary terms.<sup>2</sup> However, the latter is the form of data actually used in most, if not all, growth accounting exercises.

This paper has two purposes. The first one is to provide a critical evaluation of what superficially seems might be a useful methodological procedure; that is, to derive the rate of TFPG directly from the NIPA, as suggested by Barro and Hsieh. The second purpose is to consider, by using a simulation exercise, whether or not the rate of total factor productivity growth using aggregate data (in constant-price money terms) is always a reasonable approximation to the true rate of technical progress calculated using disaggregated data (in physical terms). We find, in fact, that the two values can differ markedly. We conclude that the growth accounting approach as presently carried out in most cases may give very misleading estimates of technical change.

### The NIPA and Growth Accounting

The thesis of Barro–Hsieh is that growth accounting exercises can be directly performed using data from the NIPA, according to which value added equals the payments to the factors of production, namely:

$$V_t \equiv w_t L_t + r_t J_t \quad (1)$$

without any other assumptions. For example, Hsieh (2002, p. 505) is of the opinion that ‘with only the *condition* that output equals factor incomes, we have the result that the primal and dual measures of the Solow residual are equal. No other assumptions are needed for this result: we do not need any assumption about the form of the production function, bias of technological change, or relationship between factor prices and their social marginal products.’ Barro (1999, p. 123) concurs: ‘the dual approach can be derived readily from the equality between output and factor income’. To show this, he writes the income accounting identity, differentiates it, and expresses it in terms of growth rates (Barro, 1999, equations (7) and (8), p. 123). Barro and Hsieh agree that, in Barro’s words, ‘it is important to recognize that the derivation of equation (8) [the growth accounting equation in Barro’s paper] uses only the condition  $V_t = r_t J_t + w_t L_t$ . No assumptions were made about the relations of factor prices to social marginal products or about the form of the production function’ (Barro, 1999, p. 123).<sup>3</sup> Barro continues: ‘If the condition  $V_t = r_t J_t + w_t L_t$  holds, then the primal and dual estimates of TFP growth inevitably coincide ... . If the condition  $V_t = r_t J_t + w_t L_t$  holds, then the discrepancy between the primal and dual estimates of TFP has to reflect the use of different data in the two calculations’ (Barro 1999, pp. 123–124).

Alternatively, from our point of view, it is important to emphasize that the symbol ‘ $\equiv$ ’ in equation (1), which defines value added, indicates that it is an accounting identity, which has, *per se*, no behavioural implications and will hold irrespective of the state of competition and whether or not an aggregate production function actually exists.

Why do Hsieh and Barro argue that growth accounting can be undertaken by simply manipulating the NIPA identity? The answer is that if equation (1) is differentiated with respect to time and expressed in growth rates, the following is obtained:

$$\hat{V}_t \equiv a_t \hat{w}_t + (1 - a_t) \hat{r}_t + a_t \hat{L}_t + (1 - a_t) \hat{J}_t \quad (2)$$

where the 'hat' denotes a proportional growth rate,  $a_t \equiv (w_t L_t) / V_t$  is the share of labour in output, and  $(1 - a_t) \equiv (r_t J_t) / V_t$  is the share of capital. Equation (2) can be rearranged to give:

$$TFPG_t \equiv \hat{V}_t - a_t \hat{L}_t - (1 - a_t) \hat{J}_t \quad (3a)$$

$$\equiv a_t \hat{w}_t + (1 - a_t) \hat{r}_t \quad (3b)$$

where *TFPG* denotes the rate of total factor productivity growth.

Equations (3a) and (3b), under the usual neoclassical assumptions, are formally equivalent to the growth accounting residual measure of *TFPG*. These assumptions include the existence of a well-behaved aggregate production function, perfectly competitive markets, Hicks-neutral technical change, and the applicability of the marginal productivity theory of factor pricing. Indeed, it may be shown that the first-order conditions from differentiating the production function will, through Euler's theorem, ensure that the accounting identity holds. A result is that the factor shares will equal the corresponding output elasticities, so we have:

$$\hat{V}_t = TFPG_t + \alpha_t \hat{L}_t + \beta_t \hat{J}_t \quad (4)$$

where  $\alpha_t = a_t$  and  $\beta_t = (1 - a_t)$  and *TFPG* is the *Solow residual*.

The underlying assumptions, however, are not innocuous. The interpretation of  $TFPG_t$  in equations (3a) and (3b) as the rate of technological progress follows directly from a comparison with equation (4). If this were not the case, there are no grounds for referring to  $TFPG_t$  in equations (3a) and (3b) as the rate of technical progress. In fact, it has been argued that the aggregate production function, through the usual neoclassical assumptions and Euler's theorem provides a theory of the income side of the NIPA (Hulten, 2000, p. 8; Prescott, 1998, p. 532; see also Jorgenson & Griliches, 1967, pp. 252–253). The theory implies that given an aggregate production  $V = f(L, J, t)$  the following equation holds:  $V = f_L L + f_J J$ . From the first-order conditions  $f_L = (\partial F / \partial L) = w$  and  $f_J = (\partial F / \partial J) = v$ , it follows that  $V = wL + vJ$  (where  $v$  is the rental price of capital) which is taken to be the accounting identity, analogous to equation (1). The neoclassical framework considers that the production function, through Euler's theorem, implies the identity. The important thing to note is that under this argument, the interpretation of *TFPG* in equations (3a) and (3b) as a measure of technical progress, or the growth of output not explained by the growth of factor inputs, holds if, and only if, the theory from which it was derived holds.

However, our argument is that the wage bill and total profits can be divided into the respective products (i.e. equation (1)) without recourse to any theory and does not require that wage and profit rates equal their respective marginal products. We maintain that the identity  $V \equiv W + \Pi \equiv wL + rJ$  holds irrespective of whether or not any of these conditions are fulfilled (Felipe & McCombie, 2003). Hsieh's quotation cited above that 'we do not need any assumption about the form of the production function, bias of technological change, or relationship between factor prices and their social marginal products' is misleading to the extent that it could be interpreted as implying that none of these conditions are required for growth

accounting. What it simply means is that if we have an identity  $X \equiv Y + Z$ , then this may be expressed as  $\hat{Y} \equiv (1/\theta)\hat{X} - ((1-\theta)/\theta)\hat{Z}$  where  $\theta \equiv Y/X$ . If we term the RHS the 'primal', then by definition it must equal the LHS, whether it is called the 'dual' or anything else. Therefore, any measurement error on either the RHS or the LHS of the equation must lead to an equal error on the opposite side. However, this should not lead us to overlook the necessary assumptions that underlie the growth accounting approach if either the dual or the primal is to be interpreted as a measure of technical progress, or of the rate of increase in efficiency.

Consequently, bearing this qualification in mind, from the accounting identity, if a consistent data set (i.e. one for which the identity (1) holds) is used for calculating the dual and the primal measures of *TFPG*, then, by definition, they must be equal. The identity consists of five variables, namely,  $V$ ,  $w$ ,  $L$ ,  $r$  and  $J$ . If values of each are obtained independently then it is possible that the identity, equation (1), will not hold because of measurement error. For this reason, to ensure consistency, one of the variables must be obtained residually. Normally, the *ex post* rate of profit is the variable that is calculated residually as  $r_t \equiv (V_t - w_t L_t)/J_t$ .<sup>4</sup>

It is important to mention that, in his calculations, Hsieh did not use the accounting identity exactly as described above. Instead of calculating  $r$  residually, he computed the rental price of capital ( $v$ ) as  $v_t = q_t [\rho_t + \delta_t - \dot{q}_t]$  (Jorgenson, 1963) where  $q$  is the price of capital,  $\rho$  is a measure of the cost of capital,  $\delta$  is the depreciation rate and  $\dot{q}$ , which equals  $dq/dt$ , is the capital gain or loss. If this is used in the identity with the other variables independently measured, then there may be a statistical or measurement problem along the lines outlined above. Nevertheless, in Felipe & McCombie (2006, 2007), we show that this issue does not pose a conceptual problem for our argument. In this case, value added should be adjusted to ensure that the identity holds.

A different approach by Hart (1996) would initially seem to agree with our arguments; but there are significant differences. He starts from the accounting identity and correctly emphasizes that 'accounting identities, unlike the usual econometric estimates of production functions, hold in equilibrium and in disequilibrium' (Hart, 1996, p. 226). He views the identity as representing the trading accounts of the representative firm, but because of data limitations, he uses macroeconomic data in his analysis. However, he calculates *TFPG*, or multifactor productivity (MFP) growth as he terms it, as the weighted growth of factor prices, citing in support of this the discussion of the dual by Jorgenson & Griliches (1967). Nevertheless, somewhat confusingly, he argues that the accounting identity (dual) approach makes it easier to explain the effects on MFP growth of 'disequilibrium, market imperfections, trade-union cost pushes, OPEC oil-price squeezes, Government price controls and incomes policies, regulation and deregulation and of other forces which influence profit margins. It must be remembered that these dual measures of MFP are purely statistical, and, so far no use has been made of production functions, returns to scale, marginal productivity, competitive equilibrium conditions or any economic theory of production' (Hart, 1996, p. 228).

What then is the justification for using factor shares as weights for the growth of the factor inputs, as Hart does? The standard justification rests solely on the argument that they are a good approximation to the output elasticities. However, this requires standard neoclassical production theory. Hart (1996, p. 229) gives an example where an increase in value added  $V$  goes entirely to labour owing to trade union bargaining: 'so the direction of causation is from left to right', i.e. from the

increase in  $V$  to the increase in  $w$ . This means that the share of wages increases, but we would argue this may have nothing to do with changes in the output elasticity of labour or in the technical conditions of production.<sup>5</sup> Thus, the shares could merely reflect the bargaining power of labour and capital and, as we have argued above, can change without there necessarily being any change in the technical conditions of production. Hence, in this case, the shares cannot be taken as reflecting the aggregate output elasticities, even if these exist.

To discuss the decomposition of output growth in any meaningful way into *TFPG* and the contribution of the growth of factor inputs *does* require aggregate production theory, *pace* Hart. This is true whether we measure *TFPG* in terms of the weighted growth of factor prices or in terms of the growth of output minus the weighted growth of factor inputs. Indeed, the two methods will, as we have seen, give identical results if  $r$  (or  $J$ ) is calculated residually (as Hart does). Any possible differences, as we have noted, will be purely statistical, owing to different data sources, etc., if the five variables are calculated independently. Indeed, Hart's empirical analysis comes largely to this conclusion and ironically his method, if not his interpretation, would be the same procedure as that adopted by a neoclassical growth accountant working from the dual.

Hart (1996) also argues that when one uses gross output and double deflation, the identity no longer holds. 'Instead we write  $Y = f(L, M, K)$  and we use production theory' (Hart, 1996, p. 229), where  $Y$  is gross output,  $M$  is materials and  $K$  is the capital stock. It is difficult to see any justification for this, as one needs to construct an implicit deflator for wages and profits and this should (must) preserve the identity in real terms. Hart concedes that analysis using the production function also has to use deflated monetary values 'so the problems created by relative changes in output and input prices are still present' (Hart, 1996, p. 229). So, according to this argument, an index number problem necessitates a particular theory, which is then subject to the same problem. Index number problems do present problems for quantitative analysis, but they are only of second-order importance in this context.

Finally, it should be noted that estimates of *TFPG* have recently been calculated using physical data for output at the seven-digit SIC product classification (Foster *et al.*, 2005). An example of this type of homogeneous output is ready-mixed concrete or boxes. However, this approach does not avoid our strictures, as the inputs are still measured in monetary terms and the weights are the cost shares. Thus, total factor productivity growth in value terms is given by:

$$TFPG_{it} = (\hat{Q}_{it} + \hat{P}_{it}) - a_{L_{it}} \hat{L}_{it} - a_{J_{it}} \hat{J}_{it} - a_{M_{it}} \hat{M}_{it} - a_{E_{it}} \hat{E}_{it} \quad (5)$$

where  $\hat{Q}$  is the growth of firm  $i$ 's physical output,  $\hat{P}_{it}$  is the rate of change of firm  $i$ 's *relative* price and  $\hat{M}$  and  $\hat{E}$  are the growth rates of materials and energy measured in monetary terms at constant prices. The  $a$ 's are the relevant factor shares. The growth of putative physical total factor productivity is given by  $TFPG - Q_{it} = TFPG_{it} - \hat{P}_{it}$ . While this approach has the advantage of removing from the estimate of total factor productivity growth fluctuations in a firm's relative price as a result of, for example, idiosyncratic demand shocks, it can be seen that the contribution to output growth of the inputs is still calculated using monetary data.

Before concluding this section, it is worth noting that, as indicated above, a crucial assumption underlying this whole approach is that a well-behaved aggregate production function exists, which should theoretically be estimated using physical quantities. This is taken to be self-evident and is rarely, if ever, explicitly

discussed.<sup>6</sup> However, this is not an innocuous assumption and indeed there are two theoretical reasons for believing that the aggregate production function cannot exist. The first stems from the results of the Cambridge Capital Theory Controversies, which seem to have been long forgotten and are rarely now mentioned in the literature. Nevertheless, however inconvenient for the aggregate neoclassical approach, these results still stand. See, for example, the recent retrospective view by Cohen & Harcourt (2003). Second, there are the equally, if not more damaging aggregation problems, recently surveyed by Felipe & Fisher (2003).

Suffice to say here that both these critiques addressed, from very different points of view, the question of whether or not the aggregate production function with the usual neoclassical properties is a legitimate concept, even when viewed as an approximation. Both criticisms come to the same conclusion on this point: the existence, and hence the use, of aggregate production functions is very problematic. This implies that the connection made in neoclassical macroeconomics between the identity, the aggregate production function and Euler's theorem is very tenuous, to say the least. Indeed, this line of reasoning is untenable if the aggregate production function does not exist;<sup>7</sup> and as Fisher pointed out, 'If aggregate capital does not exist, then of course one cannot believe in the marginal productivity of *aggregate capital*' (Fisher 1971a, p. 405; italics in the original).

Why then is the aggregate production function still so widely used? The answer, although it is rarely explicitly stated, must be that ever since the early estimations of Douglas and his colleagues, the statistical estimation of aggregate production functions gives good fits with the estimates of the output elasticities close to the observed factor shares, although time-series results are known for their fragility (see Sylos-Labini, 1995). This argument relies on Friedman's (1953) instrumental methodological justification that what matters are the successful predictions of a theory, regardless of the unrealism of the underlying assumptions. Not only is this view no longer widely held by philosophers of science, but also our argument shows that this justification is flawed, even on its own terms (McCombie, 1998a). The good statistical fit of a putative aggregate production has no implications for whether or not the aggregate production function actually exists in the sense of reflecting the underlying technology of the economy or industry.

### Some Simulation Experiments

To illustrate the above arguments, we undertook some simple simulations with a view to determining what growth accounting is actually measuring.<sup>8</sup> The key question behind the exercise is the following: if an aggregate production function does not exist, what is actually being measured as total factor productivity growth in exercises that use aggregate data? This question has the following implications: (i) if the answer is that it is some average of individual firms' productivities, then does the aggregate method accurately measure such an average? If not, why not?; and (ii), more generally, how and to what extent may we be misled by the results of aggregate growth accounting?

It should be emphasized that we are not performing Monte Carlo simulations. The purpose of this exercise is merely to generate a data set whose underlying structure is known, in both physical and monetary terms, in order to highlight the different results that the two types of data lead to in growth accounting exercises. Monte Carlo simulations studying related issues, but answering different

questions, were performed by Felipe & Holz (2001). See also Fisher (1971b) and Shaikh (1980).

Table 1 summarizes the assumptions of the simulations. We assume that there are well-defined firm micro-production functions, which are specified in physical terms, as ideally they should be. The constant price monetary value of output was calculated through a mark-up on unit labour costs and the value of total profits was generated residually through the NIPA identity, i.e. equation (1). It is assumed that each firm produces a homogenous output, which may, or may not, be the same for all firms. The analysis does not depend on this assumption so long as, in the former case, it is not possible to recover the physical quantities from the monetary value data. In the latter case, i.e. output is not homogeneous across firms, we cannot, of course, estimate a cross-section production function using physical data. It is important to emphasize that we generated two types of data, one in physical terms and the other in monetary terms. The former are constructed to give a good fit to the firm Cobb–Douglas functions (Cobb & Douglas 1928). The investigator does not know this form of the production function. Monetary data, as indicated above, are generated through the accounting identity. The investigator knows all the monetary data, but cannot recover the physical data from them.

It should also be noted that in the case where a cross-firm production function is estimated, no aggregation problem *à la* Fisher is involved if we assume all firms

**Table 1.** Summary of the assumptions of the simulations

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*Cross-Firm Estimation of the Production Function*

- There are 10 firms, one period.
- Identical production functions are of the form  $Q_i = A_0 L_i^\alpha K_i^{(1-\alpha)}$ , where  $Q_i$  and  $K_i$  are generated as random variables.  $L_i$  is calculated through the production function. These are physical data.  $A_0$  is the same across firms and normalized to 1.
- Output elasticities are (i) labour,  $\alpha = 0.25$ ; (ii) capital,  $(1-\alpha) = 0.75$ , with a random error to avoid multicollinearity.
- Value data: Firms set prices as a mark-up on unit labour costs, i.e.  $p_i = (1+\mu)w_i L_i / Q_i$  where  $\mu = 0.333$  and is the same across firms.
- Money wage rate is  $w_i = w$  and is the same across firms.
- Profit rate is  $r_i = r = 0.10$  and is the same across firms.
- Output in value terms is  $V_i = p_i Q_i$ .
- Capital stock in value terms is  $J_i = (V_i - w_i L_i) / r_i = (V_i - w_i L_i) / 0.1$ .
- Labour share in value terms is  $\alpha_i = (w_i L_i) / V_i = 1 / (1+\mu) = 0.75$ .
- Capital share in value terms is  $(1-\alpha_i) = \mu / (1+\mu) = 0.25$ .
- Mean of  $\alpha_i$  is 0.744 (range from 0.698 to 0.795)

*Rate of technical progress and TFPG*

- Outputs of the 10 firms grow at different rates over the period, but  $\hat{Q}_i = \hat{K}_i$ .
- Same rate of technical progress for all firms,  $\varphi_i = \alpha(\hat{Q}_i - \hat{L}_i)$ , assumed to be  $0.5\% = 0.25(\hat{Q}_i - \hat{L}_i)$ .
- Growth of employment  $\hat{L}_i = \hat{Q}_i - (\varphi_i / \alpha)$ .
- Output elasticities (physical terms) and average shares (value terms) are, labour,  $\alpha = 0.25$ ,  $a = 0.75$  and capital,  $(1-\alpha) = 0.75$ ,  $(1-a) = 0.25$ .
- True rate of technical progress (firm level):  $\varphi_i = \hat{Q}_i - 0.25\hat{L}_i - 0.75\hat{K}_i$ .
- Total factor productivity growth:  $TFPG_i = \hat{V}_i - 0.75\hat{L}_i - 0.25\hat{J}_i$ .

*Increasing Returns to Scale*

- $Q$ ,  $L$  and  $K$  and their growth rates are as above.
  - Output elasticities are labour,  $\alpha = 0.3$ , and capital,  $\beta = 0.9$ . Degree of returns to scale = 1.2.
  - Value data calculated as before and mark-up  $\mu$  is again 0.33.
  - Factor shares are labour,  $a = 0.75$ , and capital,  $(1-a) = 0.25$ .
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have the same technology, produce the same output and the firm is the unit of observation. This is an important point. If we need to estimate a production function using outputs and inputs summed over different firms, we encounter all the well-known aggregation problems. As monetary data have to be used in estimating this aggregate production function, we can explain why regressions using these data give a good fit to the aggregate data when theoretically they should not (Fisher, 1971b; Felipe & McCombie, 2005). However, the problem is even more fundamental than this. As will be demonstrated, the accounting identity presents insurmountable problems of interpretation, even when there are no aggregation problems of any kind regarding functional forms, or affecting output, labour or capital (see Felipe & Fisher, 2003) or problems of the type discussed in the context of the Cambridge debates regarding the nature and construction of capital stocks (Cohen & Harcourt, 2003).

The important aspect of our simulations is that they show how the use of monetary data can give results at variance with the true magnitudes of the underlying production functions and, therefore, misleading numerical estimates of both the parameters of the production function and of the 'rate of technical progress'. For clarity, we will confine the term 'technical progress' to that calculated using physical data; and the term 'total factor productivity growth' to that calculated using monetary data.

#### *Cross-Firm Estimation of the Production Function*

In the first example, data in physical units were generated for 10 firms for one period under the assumption that they all have identical Cobb–Douglas constant-returns-to-scale production functions given by:

$$Q_i = A_0 L_i^\alpha K_i^{(1-\alpha)} \quad (6)$$

where  $Q_i$  is the number of units of homogeneous output, generated as a random variable;  $K_i$  is the number of identical machines which are specific to the particular industry, also generated as a random variable;  $L_i$  is the level of labour input, generated through the production function;  $A_0$  takes the same value for all firms and was normalized to unity. The parameters  $\alpha$  and  $(1 - \alpha)$  are the output elasticities of labour and capital, respectively, and are constructed to take values of 0.25 and 0.75.<sup>9</sup> The output elasticities were thus deliberately chosen to be the converse of the factor shares found in the NIPA.

In order to generate the monetary values, each firm sets prices as a mark-up on unit labour costs, i.e.

$$p_i = (1 + \mu) w_i L_i / Q_i \quad (7)$$

The mark-up  $\mu$  is the same for all firms and takes a value of one third, so  $(1 + \mu) = 1.333$ . The wage rate is the same across firms, as is the profit rate  $r$ , which takes a value of 0.10. The monetary value of the capital stock was calculated residually through the accounting identity, equation (1), as  $J_i \equiv (V_i - wL_i)/r$ , where  $V_i$  is value added, constructed as  $V_i = p_i Q_i$  for each firm using equation (7). The values of the factor shares are directly calculated using these monetary data. Labour's share is calculated as  $a_i = (wL_i)/V_i$  and capital's share as  $(1 - a_i) = (rJ_i)/V_i$ . It should also be noted that  $a_i = 1/(1 + \mu)$ , and so it takes a value of 0.75 for each firm, with a small



variation owing to the error term added. The mean value of labour's share for the 10 firms is 0.744 (with a range of 0.698–0.795).

The mean value of the capital–output ratios in monetary terms ( $J_i/V_i$ ) is 2.57 with a range of 2.24–3.18. These values are very close to what are observed empirically, and are the result of a roughly constant rate of profit and a constant factor share of capital. As  $J/V = (1 - a)/r$ , where  $(1 - a)$  is capital's share and, as noted above, is approximately equal to 0.25 and the profit rate is 0.10, the capital–output ratio will not differ much from 2.50. As we are dealing with individual firms and we design the simulations, we know both the physical data and the monetary values, as we know the prices. However, let us assume that the prices are unknown to the investigator, as is usually the case, because the output and capital stocks for different firms are aggregated in the NIPA, etc., using monetary values. Consequently,  $V$  and  $J$  (in constant prices, although because we only have one period, the distinction between current and constant prices does not arise) were taken as proxies for  $Q$  and  $K$ .

These monetary data were then used to estimate a cross-firm production function. The results of the estimation are:

$$\ln V = 2.867 + 0.750 \ln L + 0.250 \ln J \quad \bar{R}^2 = 0.999$$

$$(478.77) \quad (136.40) \quad (45.41) \quad \text{s.e.r.} = 0.0025$$

This gives a remarkably close fit to the Cobb–Douglas production function, which is to be expected given the method used to construct the data. However, some of Douglas and his colleagues' early studies, which used real, as opposed to simulated, cross-sectional (industry or region) data, also found very close statistical fits.<sup>10</sup> The sum of the estimated coefficients is 1.00 and this is not significantly different from unity (the value of the  $t$ -ratio testing this hypothesis is 0.02). With the close correspondence between the supposed 'output elasticities' and factor shares calculated from the data (0.750; 0.744 and 0.250; 0.256), it is little wonder that such results could be interpreted as providing evidence in favour of competitive markets and disproving the Marxian argument, as Douglas (1976, p. 914) claimed (See McCombie, 1998b and Felipe & Adams, 2005).

This is notwithstanding the fact that factors are *not* paid their marginal products in physical terms in our simulation data. Competition could force firms to be  $x$ -efficient so that firms do hire the factors of production up to the point where their physical returns equal their factor rewards in terms of the commodity produced. This would determine, *inter alia*, the optimal  $L_i/Q$ , which is used in the mark-up pricing equation. However, using monetary data would still give estimates of the 'output elasticities' equal to  $1/(1 + \mu)$  and  $\mu/(1 + \mu)$ , respectively.

It should be emphasized that the estimated 'output elasticities' are, of course, not the same as the 'true' output elasticities of the micro-production function. In other words, the true output elasticity of labour is 0.25, but the estimate using monetary data is 0.75.

The goodness of fit is dependent upon the degree of variation in the mark-up. With identical mark-ups, the fit is exact (and estimation is not possible because of perfect multicollinearity). Indeed, it is the constant mark-up that is solely responsible for generating the spurious Cobb–Douglas relationship. To demonstrate this, the physical values of the three series  $Q$ ,  $L$ , and  $K$  were next generated as random numbers.  $V$  and  $J$  were calculated as before. The estimation yielded a very good fit to the Cobb–Douglas with the values of the 'output elasticities' the same as before

(the result is not reported here). This does not necessarily mean that we are postulating that that output is actually a random function of the inputs. However, when one considers the complex production processes of any modern firm, there may be some individual parts of the process subject to fixed coefficients, whereas others are subject to differing elasticities of substitution, to say nothing of differences between plants in managerial and technical efficiencies. Thus, the randomness may simply be a reflection of the severe misspecification error inherent in specifying the micro-production function as a Cobb–Douglas. However, the important point to note is that even in this case, where there is no well defined micro-production function, the use of value-added data will give the impression that there exists a well-behaved aggregate Cobb–Douglas production function.

#### *Rate of Technological Progress and Total Factor Productivity Growth*

In order to calculate the growth of total factor productivity, we need the growth rates of output, capital and labour. We assumed that outputs of the 10 hypothetical firms grow at different rates (we only have one growth period), but, for expositional purposes only, the series were constructed such that the growth rate of the physical capital–output ratio is zero (i.e. output and capital grow at the same rate) for all firms. It was also assumed that each firm experiences the same rate of technical progress ( $\varphi_i$ ), namely, 0.5% per annum, which is equal to  $\varphi_i = \alpha(\hat{Q}_i - \hat{L}_i)$ . This is due to the fact that the underlying production functions are Cobb–Douglas and that the growth of output equals the growth rate of capital, where both are measured in physical terms. Hence, the growth rate of employment for each firm is given by as  $\hat{L}_i = \hat{Q}_i - (\varphi_i / \alpha)$ , where  $\varphi_i = 0.5\%$ , as noted above. The output elasticities of labour and capital in *physical* terms are again 0.25 and 0.75. However, the average factor shares in *monetary* terms are 0.745 (with a range from 0.698 to 0.795) and 0.255, and the aggregate shares are also 0.745 and 0.255, as each firm has the same mark-up. This means that the labour share of each firm is the same and if we aggregate over firms, the aggregate share comes to about the same value.

Although the investigator cannot use the values of the variables in physical terms, we are able to do so simply because we have constructed the data set. Consequently, we can calculate the rate of technical progress for each hypothetical firm separately using the standard growth accounting equation; that is,

$$\varphi_i \equiv \hat{Q}_i - a\hat{L}_i - (1-a)\hat{K}_i \quad (8)$$

where  $a$  and  $(1-a)$ , the factor shares of labour and capital, are assumed to equal the output elasticities  $\alpha$  and  $(1-\alpha)$ , 0.25 and 0.75, respectively.

As the rate of technical progress is the same for each firm, we can talk about the rate of technical progress being 0.5% per annum, even in the case where we assume that the physical outputs of the various firms are not homogeneous.

However, the individual prices of the various firms are not available to the investigator and so it is not possible to extract data on the physical units of output. All that can be used in empirical work, as is usually true in practice, is the constant price value of output and of the capital stock. The growth of total factor productivity in these circumstances is given by:

$$TFPG_i \equiv \hat{V}_i - a\hat{L}_i - (1-a)\hat{J}_i \quad (9)$$

where now the shares of labour and capital are 0.75 and 0.25, respectively.

The unweighted mean rate of total factor productivity growth of the individual firms is 1.49% per annum, which, not surprisingly, is almost identical to the rate of total factor productivity growth (1.48% per annum) obtained by aggregating the monetary data over all 10 firms and using these in an aggregate version of equation (9).

Thus, the use of physical data yields technical progress accounting, on average (the unweighted mean), for  $\varphi_i = 0.25(\hat{Q}_i - \hat{L}_i)$ , that is, 25% of labour productivity growth, with a very small difference between firms owing to the small random element introduced for the reasons noted above.<sup>11</sup> However, the use of monetary data for each of the 10 firms gives a mean value of the rate of total factor productivity that is  $TFPG_i = 0.75(\hat{V}_i - \hat{L}_i)$ , or 75% the growth of labour productivity, with a range from 80% to 70%. The figure using the aggregate data (i.e. using the aggregate values of output, labour and capital) is 74%. The reason for the marked difference between these values and the 'true' rate of technical progress is that labour's share of output in value terms is 0.75, while the 'true' output elasticity of labour is 0.25.

It is worth noting that  $TFPG$  in equations (3a) and (3b) can be written as  $TFPG_t \equiv \hat{V}_t - a_t \hat{L}_t - (1 - a_t) \hat{J}_t$  which equals  $a_t(\hat{V} - \hat{L}) + (1 - a_t)(\hat{V} - \hat{J})$ . The last expression is a weighted average of the growth rates of labour and capital productivity. Therefore, it could be argued that  $TFPG$  is an average measure of productivity growth. This interpretation faces, however, the problems discussed in this subsection, namely, that the figure computed is not equivalent to the true rate of technical progress. The growth of labour and capital productivity in monetary terms are unlikely to be the same as those measured in physical terms and the weights are arbitrary, in that the shares will change because of changes in, for example, the mark-up.

Consequently, the use of monetary data produces a significantly different estimate of the rate of technical progress, compared with the true value obtained using physical data. Even with well-defined underlying Cobb–Douglas micro-production functions expressed in physical terms, the use of monetary data as a proxy for output can give very misleading estimates of the rate of 'technical progress'.

### *Increasing Returns to Scale and Total Factor Productivity Growth*

What happens if the individual firms are subject to increasing returns to scale when physical data are used? To examine this question, we first estimated the cross-firm production functions using monetary data when the micro-production functions exhibit the same degree of increasing returns to scale. The data for the inputs in physical terms were the same as those used in the previous simulation, with the exception that now the elasticities were multiplied by 1.20, so  $\alpha' = 0.30$  and  $\beta' = 0.90$ . This represents a substantial degree of returns to scale and results in a value of output that is significantly larger than when constant returns to scale are imposed. The monetary data were calculated the same way as before, with a mark-up once again of one-third.

Estimating the unrestricted Cobb–Douglas production function gives a result that is virtually identical to that for constant returns to scale, and reported above,

except for a change in the value of the intercept. Consequently, we do not report the results here. The estimates of the putative output elasticities are once again very close to the observed monetary values of the factor shares and sum to unity, thereby erroneously suggesting that the production process of the various firms are subject to constant returns to scale. The reason for this seemingly paradoxical result is that the calculation of value added is given by  $pQ = V = (1 + \mu)wL$  and as nominal wages and the level of employment are the same as before, so is the constant price measure of value added, although the price per unit is now lower. Recall that we showed above that with monetary data estimation of the production function would automatically yield estimates of the 'output elasticities' equal to those of the factor shares.

Next, we calculated the rate of technical progress and the growth of total factor productivity. For comparability with the constant returns to scale case, the growth rates of physical output, capital and labour were the same as before. The rate of technical progress was calculated using the physical data as

$$\varphi_i = \hat{Q}_i - \alpha' \hat{L}_i - \beta' \hat{K}_i \quad (10)$$

where  $\alpha' = 1.2\alpha$  and  $\beta' = 1.2(1 - \alpha)$ . It can be seen that the rate of technical progress calculated using equation (10) will differ from the 0.5% in the case of constant returns to scale. In fact, it will be on average lower, given the larger weights on the growth of the factor inputs. The rate of technical progress, calculated using equation (10) for each firm now varies considerably across firms (for reasons of space we do not report the full results).<sup>12</sup> The unweighted mean is (coincidentally) 0.00% per annum, with a range of  $\pm 0.4$  percentage points per annum.<sup>13</sup>

However, the growth rates of total factor productivity of the individual industries, calculated using equation (9) and monetary data, are again all approximately 1.5% per annum. This is because the shares of labour and capital in monetary terms are once again 0.75 and 0.25, and the growth rates of  $V$ ,  $L$  and  $J$  are the same as before.<sup>14</sup> Thus, the use of monetary data can give a very misleading estimate of the true rate of technical progress. The use of monetary data erroneously ascribes the effect of increasing returns on increasing the efficiency of the factors of production to the rate of technical progress.

### Summary

The actual figures in the simulations are, of course, arbitrary as they depend on the assumed rates of growth of physical output, capital and employment. Nevertheless, we can draw some conclusions from these simulations.

- The growth of total factor productivity depends crucially on the weights attached to the growth of capital and labour. The growth accounting approach assumes that factors are paid their marginal products and hence the technologically determined output elasticities will equal the factor shares. However, when monetary data are used we have shown that the factor shares will *always* equal the putative output elasticities and both are determined by  $1/(1 + \mu)$  (for labour) and  $\mu/(1 + \mu)$  (for capital), where  $1 + \mu$ , it will be recalled, is the mark-up. The estimates of the output elasticities using monetary data will almost certainly differ from the true ones (always assuming that there is a well-defined micro-production function in physical terms in the first place).

- Where it is possible to compare the 'true' growth rate of technical progress with the growth of total factor productivity in monetary terms (namely at the firm level here), the two values will probably differ markedly. In general, it is not possible to recover the physical quantities (of both output and capital) from the monetary data through the use of individual prices and so resort is made to the use of monetary data in real terms with potentially very misleading results. Where the physical data is available, it can only be at a very low level of aggregation. It requires each output and capital good to be measured separately in physical terms.
- The problems posed by the accounting identity are independent from, and in a sense more fundamental than, either the aggregation problem or the Cambridge Capital Theory Controversies. This is because the problems arise even when the last two do not pose any problems.

It should be clear by now that from the simulations that all estimations of aggregate production functions do is to approximate the NIPA accounting identity, and as such they are exercises that, although algebraically correct, stand without any theoretical foundation.

What then are we to make of Barro's (1999, p. 123) argument that an economy that experiences an increase in both its real wage and profit rates must have increased its overall level of productivity? It could be argued that  $TFPG_t = a_t \hat{w}_t + (1 - a_t) \hat{r}_t$  measures such a rate of growth of efficiency. Certainly, under these circumstances one can say that the economy is better off, because obviously this measure contributes positively to output growth. The point to note is that, as we mentioned above, it is not possible to ascribe this unambiguously to the result of technical change the way it is done in the neoclassical model (i.e. by claiming that this concept is derived from a testable model). There is no reason to assume that factor shares, i.e. the appropriate or theoretically justified weights according to Barro (1999, p. 123), equal the output elasticities of the true aggregate production function (if it, in fact, exists), or that production is necessarily subject to constant returns to scale (although this is what the use of monetary data will show). The derivation is simply a tautology resulting from an identity with no behavioural assumptions or implications, except that factor shares in monetary terms are constant.<sup>15</sup>

Suppose, for the sake of the argument, that the factor shares are determined by the relative bargaining power of labour *vis-à-vis* capital. Let us assume further that capital's share increases and labour's share falls as a result of changes in legislation relating to trade unions. Even though in physical terms the growth rates of output and inputs, and hence of technical change, remain the same by assumption, the calculation of *TFPG* will show a decline compared with its previous value. Hence, the concept of total factor productivity growth cannot be given a technological interpretation.

## Conclusions

This paper has shown how the use of monetary data may give very misleading estimates of the true rate of technical progress. The simulation exercises have shown how the use of monetary data will give erroneous estimates of the sources of growth. For example, even though the micro-production functions each exhibited increasing returns to scale with labour and capital's output elasticities taking a value of 0.3 and 0.9, respectively, empirical estimation with monetary data,

however, gives the traditional values of 0.75 and 0.25. Moreover, the estimates of *TFPG* are very different from the true values at the industry level.

How are then we to interpret the Solow residual, in either its primal or dual form? For it to be seen as a uniquely determined measure of technical progress requires the assumption that the relationship between the inputs and output of the economy (or individual industries) can be represented by an aggregate production function. This seems implausible, given the theoretical objections that have been levelled against it. As Hahn (1972, cited by Blaug, 1974, p. 19) put it:

It has often been the case that a neo-classical theory has been attempted in terms of aggregate production functions and aggregates like capital. Except under absurdly unrealistic assumptions such an aggregate theory cannot be shown to follow from the proper theory and in general is therefore open to severe logical objection. ... It is extremely unfortunate, and to some extent the fault of its practitioners, that neo-classical theory has come to be identified with this aggregation theory. On purely theoretical grounds there is nothing to be said in its favor. The view that that nonetheless it 'may work in practice' sounds a little bogus and in any case the onus of proof is on those who maintain this.

Recourse to empirical estimation does not shed any light on the issue, and, indeed, it is always possible to get a perfect fit to the data in a form that resembles an aggregate production function.

Barro (1999) has suggested how the growth accounting approach may be adjusted to incorporate the arguments of the endogenous growth models. However, this approach also requires the existence of a well-behaved aggregate production function. The problem is that physical data for industrial processes are not readily available and where it is, it is at a very low level of aggregation. Moreover, it is very unlikely that the simple (if not simplistic) Cobb–Douglas, or even the more flexible functional forms such as the translog, are likely adequately to model the production processes of a modern manufacturing plant or a firm in the service sector.

Solow once remarked that: 'I have never thought of the macroeconomic production function as a rigorous justifiable concept. In my mind, it is either an illuminating parable, or else a mere device for handling data, to be used so long as it gives good empirical results, and to be abandoned as soon as it does not, or as soon as something better comes along' (Solow 1966, pp. 1259–1260). Perhaps such a time has arrived. Felipe & Fisher (2003, p. 257) concluded their survey on the aggregation literature as follows: 'Macroeconomists should pause before continuing to do applied work with no sound foundation and dedicate some time to studying other approaches to value, distribution, employment, growth, technical progress, etc., in order to understand which questions can legitimately be posed to the empirical aggregate data.' The problems associated with the interpretation of neoclassical growth accounting exercise merely serves to reinforce these conclusions.

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## Notes

1. The national accounts only give data for total wages ( $W$ ) and total profits ( $\Pi$ ) but employment and capital stocks (calculated using the perpetual inventory method) are readily available from many statistical sources such as the OECD and the US Bureau of Labour Statistics.
2. This criticism, in a very sketchy form, can be traced back to the Reder (1943). More recent expositions include Phelps Brown (1957), Shaikh (1980) and Simon (1979).
3. The notation in Barro's quotations has been changed to make it consistent with that in this paper.
4. Alternatively, the values of one or more of the variables must be adjusted to ensure that the identity holds.
5. Hart suggests that this may possibly be due to a reorganization of labour agreed to by the unions. To the extent that this leads to increased efficiency and output, then the causation could be said to run from right to left.
6. Reder (1943, p. 260) is an exception: 'in economic theory a production function relates *physical* quantities of output to quantities of *physical* input of the factors. But the Cobb-Douglas function ... relates the *value* added in manufacturing, defined as product, to the physical input of labor services (labor) and the value of plant and equipment owned (capital)' (italics in the original).
7. As a consequence, parameters such as the aggregate elasticity of substitution are meaningless. In the words of Fisher *et al.* (1977): 'the elasticity of substitution in these production functions is an "estimate" of nothing; there is no true aggregate parameter to which it corresponds' (Fisher *et al.*, 1977, p. 312).
8. These results extend an earlier simulation based on time-series data (McCombie, 2001).
9. To prevent perfect multicollinearity, a small random variable was added to these and, where necessary, other variables used in the simulation.
10. For example, Douglas (1976, p. 906) reports the following results for a production function based on American cross-section studies, 1904, 1909, 1914 and 1919.

Year	$\alpha$	$\beta$	$\alpha + \beta$
1904	0.65 (32.5)	0.31 (15.5)	0.96
1909	0.63 (31.5)	0.34 (17.0)	0.97
1914	0.61 (30.5)	0.37 (18.5)	0.98
1919	0.76 (38.0)	0.25 (12.5)	1.01
Average	0.66 (33.0)	0.32 (16.0)	0.98

Notes:  $t$ -values in parentheses. Total number of observations: 1490.

11. This is because there is no growth in the physical capital-output ratio,  $\varphi_i = \alpha(\hat{Q}_i - \hat{L}_i)$  and  $\alpha$ , the physical output elasticity of labour, is equal to 0.25. Hence the rate of technical progress equals one quarter of the growth of labour productivity.
12. This is because while the growth rates of  $Q$  and  $K$  are the same between firms, employment growth rates differ and so the change in weighting causes the rate of technical change now to differ across firms.
13. As we cannot sum across the physical quantities, we cannot calculate a meaningful average rate of technical progress, as the individual rates cannot be unambiguously or uniquely weighted. Nevertheless, we did calculate the unweighted mean.
14. With a constant mark-up of 1.333, the shares will be always 0.75 and 0.25, regardless of the technical conditions of production (e.g. the degree of returns to scale).
15. We have adopted a mark-up pricing policy to generate constant shares in value terms. However, there are any number ways that this could occur (e.g. through a bargaining model or a Kaldorian model of distribution).

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## The Tyranny of the Identity: Growth Accounting Revisited

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**ABSTRACT** *It has been argued in the literature that growth accounting may be undertaken by directly differentiating the national income and product accounts identity where total income equals labour's total compensation and total profits. This paper shows that this is simply an exercise in the manipulation of an accounting identity without necessarily having any theoretical foundation. Simulations show that the estimates of total factor productivity growth resulting from growth accounting performed with aggregate monetary data are not equivalent to the true rate of technological progress implied by the micro-data. This suggests that results from the orthodox growth accounting approach may be very misleading.*

**KEY WORDS:** Total factor productivity growth, accounting identity.

**JEL CLASSIFICATION:** O11, O16, O47, O53

### Introduction

A number of papers (Barro, 1999; Hsieh, 1999, 2002) have argued that the neoclassical growth accounting exercises, specifically the derivation of the dual measure of total factor productivity growth (TFPG), can be performed by simply differentiating the National Income and Product Accounts (NIPA) identity. The identity is  $V_t \equiv W_t + \Pi_t \equiv w_t L_t + r_t K_t$  where  $V$  is value added,  $W$  is the total wage bill,  $\Pi$  denotes total profits,  $w$  is the average wage,  $L$  is employment,  $r$  is the rate of profit and  $K$  is the value of the capital stock. The monetary, or value, variables are all measured in real terms.<sup>1</sup>

This approach to growth accounting stands in marked contrast to the arguments of Felipe & McCombie (2003). There it was argued that precisely because of the existence of the accounting identity, growth accounting exercises amount to no more than manipulations of the *ex-post* national income accounting identity, and, as such, they are tautologies without necessarily any behavioural content. The reason for the problem that Felipe & McCombie (2003) highlight is that there is an important difference between using physical data, only available

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at a disaggregated level and which should theoretically be used in growth accounting and production function studies, and data expressed in constant-price monetary terms.<sup>2</sup> However, the latter is the form of data actually used in most, if not all, growth accounting exercises.

This paper has two purposes. The first one is to provide a critical evaluation of what superficially seems might be a useful methodological procedure; that is, to derive the rate of TFPG directly from the NIPA, as suggested by Barro and Hsieh. The second purpose is to consider, by using a simulation exercise, whether or not the rate of total factor productivity growth using aggregate data (in constant-price money terms) is always a reasonable approximation to the true rate of technical progress calculated using disaggregated data (in physical terms). We find, in fact, that the two values can differ markedly. We conclude that the growth accounting approach as presently carried out in most cases may give very misleading estimates of technical change.

### The NIPA and Growth Accounting

The thesis of Barro–Hsieh is that growth accounting exercises can be directly performed using data from the NIPA, according to which value added equals the payments to the factors of production, namely:

$$V_t \equiv w_t L_t + r_t J_t \quad (1)$$

without any other assumptions. For example, Hsieh (2002, p. 505) is of the opinion that ‘with only the *condition* that output equals factor incomes, we have the result that the primal and dual measures of the Solow residual are equal. No other assumptions are needed for this result: we do not need any assumption about the form of the production function, bias of technological change, or relationship between factor prices and their social marginal products.’ Barro (1999, p. 123) concurs: ‘the dual approach can be derived readily from the equality between output and factor income’. To show this, he writes the income accounting identity, differentiates it, and expresses it in terms of growth rates (Barro, 1999, equations (7) and (8), p. 123). Barro and Hsieh agree that, in Barro’s words, ‘it is important to recognize that the derivation of equation (8) [the growth accounting equation in Barro’s paper] uses only the condition  $V_t = r_t J_t + w_t L_t$ . No assumptions were made about the relations of factor prices to social marginal products or about the form of the production function’ (Barro, 1999, p. 123).<sup>3</sup> Barro continues: ‘If the condition  $V_t = r_t J_t + w_t L_t$  holds, then the primal and dual estimates of TFP growth inevitably coincide . . . . If the condition  $V_t = r_t J_t + w_t L_t$  holds, then the discrepancy between the primal and dual estimates of TFP has to reflect the use of different data in the two calculations’ (Barro 1999, pp. 123–124).

Alternatively, from our point of view, it is important to emphasize that the symbol ‘ $\equiv$ ’ in equation (1), which defines value added, indicates that it is an accounting identity, which has, *per se*, no behavioural implications and will hold irrespective of the state of competition and whether or not an aggregate production function actually exists.

Why do Hsieh and Barro argue that growth accounting can be undertaken by simply manipulating the NIPA identity? The answer is that if equation (1) is differentiated with respect to time and expressed in growth rates, the following is obtained:

$$\hat{V}_t \equiv a_t \hat{w}_t + (1 - a_t) \hat{r}_t + a_t \hat{L}_t + (1 - a_t) \hat{J}_t \quad (2)$$

where the 'hat' denotes a proportional growth rate,  $a_t \equiv (w_t L_t) / V_t$  is the share of labour in output, and  $(1 - a_t) \equiv (r_t J_t) / V_t$  is the share of capital. Equation (2) can be rearranged to give:

$$TFPG_t \equiv \hat{V}_t - a_t \hat{L}_t - (1 - a_t) \hat{J}_t \quad (3a)$$

$$\equiv a_t \hat{w}_t + (1 - a_t) \hat{r}_t \quad (3b)$$

where *TFPG* denotes the rate of total factor productivity growth.

Equations (3a) and (3b), under the usual neoclassical assumptions, are formally equivalent to the growth accounting residual measure of *TFPG*. These assumptions include the existence of a well-behaved aggregate production function, perfectly competitive markets, Hicks-neutral technical change, and the applicability of the marginal productivity theory of factor pricing. Indeed, it may be shown that the first-order conditions from differentiating the production function will, through Euler's theorem, ensure that the accounting identity holds. A result is that the factor shares will equal the corresponding output elasticities, so we have:

$$\hat{V}_t = TFPG_t + \alpha_t \hat{L}_t + \beta_t \hat{J}_t \quad (4)$$

where  $\alpha_t = a_t$  and  $\beta_t = (1 - a_t)$  and *TFPG* is the *Solow residual*.

The underlying assumptions, however, are not innocuous. The interpretation of  $TFPG_t$  in equations (3a) and (3b) as the rate of technological progress follows directly from a comparison with equation (4). If this were not the case, there are no grounds for referring to  $TFPG_t$  in equations (3a) and (3b) as the rate of technical progress. In fact, it has been argued that the aggregate production function, through the usual neoclassical assumptions and Euler's theorem provides a theory of the income side of the NIPA (Hulten, 2000, p. 8; Prescott, 1998, p. 532; see also Jorgenson & Griliches, 1967, pp. 252-253). The theory implies that given an aggregate production  $V = f(L, J, t)$  the following equation holds:  $V = f_L L + f_J J$ . From the first-order conditions  $f_L = (\partial F / \partial L) = w$  and  $f_J = (\partial F / \partial J) = v$ , it follows that  $V = wL + vJ$  (where  $v$  is the rental price of capital) which is taken to be the accounting identity, analogous to equation (1). The neoclassical framework considers that the production function, through Euler's theorem, implies the identity. The important thing to note is that under this argument, the interpretation of *TFPG* in equations (3a) and (3b) as a measure of technical progress, or the growth of output not explained by the growth of factor inputs, holds if, and only if, the theory from which it was derived holds.

However, our argument is that the wage bill and total profits can be divided into the respective products (i.e. equation (1)) without recourse to any theory and does not require that wage and profit rates equal their respective marginal products. We maintain that the identity  $V \equiv W + \Pi \equiv wL + rJ$  holds irrespective of whether or not any of these conditions are fulfilled (Felipe & McCombie, 2003). Hsieh's quotation cited above that 'we do not need any assumption about the form of the production function, bias of technological change, or relationship between factor prices and their social marginal products' is misleading to the extent that it could be interpreted as implying that none of these conditions are required for growth

accounting. What it simply means is that if we have an identity  $X \equiv Y + Z$ , then this may be expressed as  $\hat{Y} \equiv (1/\theta)\hat{X} - ((1-\theta)/\theta)\hat{Z}$  where  $\theta \equiv Y/X$ . If we term the RHS the 'primal', then by definition it must equal the LHS, whether it is called the 'dual' or anything else. Therefore, any measurement error on either the RHS or the LHS of the equation must lead to an equal error on the opposite side. However, this should not lead us to overlook the necessary assumptions that underlie the growth accounting approach if either the dual or the primal is to be interpreted as a measure of technical progress, or of the rate of increase in efficiency.

Consequently, bearing this qualification in mind, from the accounting identity, if a consistent data set (i.e. one for which the identity (1) holds) is used for calculating the dual and the primal measures of *TFPG*, then, by definition, they must be equal. The identity consists of five variables, namely,  $V$ ,  $w$ ,  $L$ ,  $r$  and  $J$ . If values of each are obtained independently then it is possible that the identity, equation (1), will not hold because of measurement error. For this reason, to ensure consistency, one of the variables must be obtained residually. Normally, the *ex post* rate of profit is the variable that is calculated residually as  $r_t \equiv (V_t - w_t L_t)/J_t$ .<sup>4</sup>

It is important to mention that, in his calculations, Hsieh did not use the accounting identity exactly as described above. Instead of calculating  $r$  residually, he computed the rental price of capital ( $v$ ) as  $v_t = q_t [\rho_t + \delta_t - \dot{q}_t]$  (Jorgenson, 1963) where  $q$  is the price of capital,  $\rho$  is a measure of the cost of capital,  $\delta$  is the depreciation rate and  $\dot{q}$ , which equals  $dq/dt$ , is the capital gain or loss. If this is used in the identity with the other variables independently measured, then there may be a statistical or measurement problem along the lines outlined above. Nevertheless, in Felipe & McCombie (2006, 2007), we show that this issue does not pose a conceptual problem for our argument. In this case, value added should be adjusted to ensure that the identity holds.

A different approach by Hart (1996) would initially seem to agree with our arguments; but there are significant differences. He starts from the accounting identity and correctly emphasizes that 'accounting identities, unlike the usual econometric estimates of production functions, hold in equilibrium and in disequilibrium' (Hart, 1996, p. 226). He views the identity as representing the trading accounts of the representative firm, but because of data limitations, he uses macroeconomic data in his analysis. However, he calculates *TFPG*, or multifactor productivity (MFP) growth as he terms it, as the weighted growth of factor prices, citing in support of this the discussion of the dual by Jorgenson & Griliches (1967). Nevertheless, somewhat confusingly, he argues that the accounting identity (dual) approach makes it easier to explain the effects on MFP growth of 'disequilibrium, market imperfections, trade-union cost pushes, OPEC oil-price squeezes, Government price controls and incomes policies, regulation and deregulation and of other forces which influence profit margins. It must be remembered that these dual measures of MFP are purely statistical, and, so far no use has been made of production functions, returns to scale, marginal productivity, competitive equilibrium conditions or any economic theory of production' (Hart, 1996, p. 228).

What then is the justification for using factor shares as weights for the growth of the factor inputs, as Hart does? The standard justification rests solely on the argument that they are a good approximation to the output elasticities. However, this requires standard neoclassical production theory. Hart (1996, p. 229) gives an example where an increase in value added  $V$  goes entirely to labour owing to trade union bargaining: 'so the direction of causation is from left to right', i.e. from the

increase in  $V$  to the increase in  $w$ . This means that the share of wages increases, but we would argue this may have nothing to do with changes in the output elasticity of labour or in the technical conditions of production.<sup>5</sup> Thus, the shares could merely reflect the bargaining power of labour and capital and, as we have argued above, can change without there necessarily being any change in the technical conditions of production. Hence, in this case, the shares cannot be taken as reflecting the aggregate output elasticities, even if these exist.

To discuss the decomposition of output growth in any meaningful way into *TFPG* and the contribution of the growth of factor inputs *does* require aggregate production theory, *pace* Hart. This is true whether we measure *TFPG* in terms of the weighted growth of factor prices or in terms of the growth of output minus the weighted growth of factor inputs. Indeed, the two methods will, as we have seen, give identical results if  $r$  (or  $J$ ) is calculated residually (as Hart does). Any possible differences, as we have noted, will be purely statistical, owing to different data sources, etc., if the five variables are calculated independently. Indeed, Hart's empirical analysis comes largely to this conclusion and ironically his method, if not his interpretation, would be the same procedure as that adopted by a neoclassical growth accountant working from the dual.

Hart (1996) also argues that when one uses gross output and double deflation, the identity no longer holds. 'Instead we write  $Y = f(L, M, K)$  and we use production theory' (Hart, 1996, p. 229), where  $Y$  is gross output,  $M$  is materials and  $K$  is the capital stock. It is difficult to see any justification for this, as one needs to construct an implicit deflator for wages and profits and this should (must) preserve the identity in real terms. Hart concedes that analysis using the production function also has to use deflated monetary values 'so the problems created by relative changes in output and input prices are still present' (Hart, 1996, p. 229). So, according to this argument, an index number problem necessitates a particular theory, which is then subject to the same problem. Index number problems do present problems for quantitative analysis, but they are only of second-order importance in this context.

Finally, it should be noted that estimates of *TFPG* have recently been calculated using physical data for output at the seven-digit SIC product classification (Foster *et al.*, 2005). An example of this type of homogeneous output is ready-mixed concrete or boxes. However, this approach does not avoid our strictures, as the inputs are still measured in monetary terms and the weights are the cost shares. Thus, total factor productivity growth in value terms is given by:

$$TFPG_{it} = (\hat{Q}_{it} + \hat{P}_{it}) - a_{L_{it}} \hat{L}_{it} - a_{J_{it}} \hat{J}_{it} - a_{M_{it}} \hat{M}_{it} - a_{E_{it}} \hat{E}_{it} \quad (5)$$

where  $\hat{Q}$  is the growth of firm  $i$ 's physical output,  $\hat{P}_{it}$  is the rate of change of firm  $i$ 's relative price and  $\hat{M}$  and  $\hat{E}$  are the growth rates of materials and energy measured in monetary terms at constant prices. The  $a$ 's are the relevant factor shares. The growth of putative physical total factor productivity is given by  $TFPG - Q_{it} = TFPG_{it} - \hat{P}_{it}$ . While this approach has the advantage of removing from the estimate of total factor productivity growth fluctuations in a firm's relative price as a result of, for example, idiosyncratic demand shocks, it can be seen that the contribution to output growth of the inputs is still calculated using monetary data.

Before concluding this section, it is worth noting that, as indicated above, a crucial assumption underlying this whole approach is that a well-behaved aggregate production function exists, which should theoretically be estimated using physical quantities. This is taken to be self-evident and is rarely, if ever, explicitly

discussed.<sup>6</sup> However, this is not an innocuous assumption and indeed there are two theoretical reasons for believing that the aggregate production function cannot exist. The first stems from the results of the Cambridge Capital Theory Controversies, which seem to have been long forgotten and are rarely now mentioned in the literature. Nevertheless, however inconvenient for the aggregate neoclassical approach, these results still stand. See, for example, the recent retrospective view by Cohen & Harcourt (2003). Second, there are the equally, if not more damaging aggregation problems, recently surveyed by Felipe & Fisher (2003).

Suffice to say here that both these critiques addressed, from very different points of view, the question of whether or not the aggregate production function with the usual neoclassical properties is a legitimate concept, even when viewed as an approximation. Both criticisms come to the same conclusion on this point: the existence, and hence the use, of aggregate production functions is very problematic. This implies that the connection made in neoclassical macroeconomics between the identity, the aggregate production function and Euler's theorem is very tenuous, to say the least. Indeed, this line of reasoning is untenable if the aggregate production function does not exist;<sup>7</sup> and as Fisher pointed out, 'If aggregate capital does not exist, then of course one cannot believe in the marginal productivity of *aggregate capital*' (Fisher 1971a, p. 405; italics in the original).

Why then is the aggregate production function still so widely used? The answer, although it is rarely explicitly stated, must be that ever since the early estimations of Douglas and his colleagues, the statistical estimation of aggregate production functions gives good fits with the estimates of the output elasticities close to the observed factor shares, although time-series results are known for their fragility (see Sylos-Labini, 1995). This argument relies on Friedman's (1953) instrumental methodological justification that what matters are the successful predictions of a theory, regardless of the unrealism of the underlying assumptions. Not only is this view no longer widely held by philosophers of science, but also our argument shows that this justification is flawed, even on its own terms (McCombie, 1998a). The good statistical fit of a putative aggregate production has no implications for whether or not the aggregate production function actually exists in the sense of reflecting the underlying technology of the economy or industry.

### Some Simulation Experiments

To illustrate the above arguments, we undertook some simple simulations with a view to determining what growth accounting is actually measuring.<sup>8</sup> The key question behind the exercise is the following: if an aggregate production function does not exist, what is actually being measured as total factor productivity growth in exercises that use aggregate data? This question has the following implications: (i) if the answer is that it is some average of individual firms' productivities, then does the aggregate method accurately measure such an average? If not, why not?; and (ii), more generally, how and to what extent may we be misled by the results of aggregate growth accounting?

It should be emphasized that we are not performing Monte Carlo simulations. The purpose of this exercise is merely to generate a data set whose underlying structure is known, in both physical and monetary terms, in order to highlight the different results that the two types of data lead to in growth accounting exercises. Monte Carlo simulations studying related issues, but answering different

questions, were performed by Felipe & Holz (2001). See also Fisher (1971b) and Shaikh (1980).

Table 1 summarizes the assumptions of the simulations. We assume that there are well-defined firm micro-production functions, which are specified in physical terms, as ideally they should be. The constant price monetary value of output was calculated through a mark-up on unit labour costs and the value of total profits was generated residually through the NIPA identity, i.e. equation (1). It is assumed that each firm produces a homogenous output, which may, or may not, be the same for all firms. The analysis does not depend on this assumption so long as, in the former case, it is not possible to recover the physical quantities from the monetary value data. In the latter case, i.e. output is not homogeneous across firms, we cannot, of course, estimate a cross-section production function using physical data. It is important to emphasize that we generated two types of data, one in physical terms and the other in monetary terms. The former are constructed to give a good fit to the firm Cobb–Douglas functions (Cobb & Douglas 1928). The investigator does not know this form of the production function. Monetary data, as indicated above, are generated through the accounting identity. The investigator knows all the monetary data, but cannot recover the physical data from them.

It should also be noted that in the case where a cross-firm production function is estimated, no aggregation problem *à la* Fisher is involved if we assume all firms

**Table 1.** Summary of the assumptions of the simulations

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*Cross-Firm Estimation of the Production Function*

- There are 10 firms, one period.
- Identical production functions are of the form  $Q_i = A_0 L_i^\alpha K_i^{(1-\alpha)}$ , where  $Q_i$  and  $K_i$  are generated as random variables.  $L_i$  is calculated through the production function. These are physical data.  $A_0$  is the same across firms and normalized to 1.
- Output elasticities are (i) labour,  $\alpha = 0.25$ ; (ii) capital,  $(1-\alpha) = 0.75$ , with a random error to avoid multicollinearity.
- Value data: Firms set prices as a mark-up on unit labour costs, i.e.  $p_i = (1+\mu)w_i L_i / Q_i$  where  $\mu = 0.333$  and is the same across firms.
- Money wage rate is  $w_i = w$  and is the same across firms.
- Profit rate is  $r_i = r = 0.10$  and is the same across firms.
- Output in value terms is  $V_i = p_i Q_i$ .
- Capital stock in value terms is  $J_i = (V_i - w_i L_i) / r_i = (V_i - w L_i) / 0.1$ .
- Labour share in value terms is  $\alpha_i = (w L_i) / V_i = 1 / (1+\mu) = 0.75$ .
- Capital share in value terms is  $(1-\alpha_i) = \mu / (1+\mu) = 0.25$ .
- Mean of  $\alpha_i$  is 0.744 (range from 0.698 to 0.795)

*Rate of technical progress and TFPG*

- Outputs of the 10 firms grow at different rates over the period, but  $\hat{Q}_i = \hat{K}_i$ .
- Same rate of technical progress for all firms,  $\varphi_i = \alpha(\hat{Q}_i - \hat{L}_i)$ , assumed to be  $0.5\% = 0.25(\hat{Q}_i - \hat{L}_i)$ .
- Growth of employment  $\hat{L}_i = \hat{Q}_i - (\varphi_i / \alpha)$ .
- Output elasticities (physical terms) and average shares (value terms) are, labour,  $\alpha = 0.25$ ,  $a = 0.75$  and capital,  $(1-\alpha) = 0.75$ ,  $(1-a) = 0.25$ .
- True rate of technical progress (firm level):  $\varphi_i = \hat{Q}_i - 0.25\hat{L}_i - 0.75\hat{K}_i$ .
- Total factor productivity growth:  $TFPG_i = \hat{V}_i - 0.75\hat{L}_i - 0.25\hat{J}_i$ .

*Increasing Returns to Scale*

- $Q$ ,  $L$  and  $K$  and their growth rates are as above.
  - Output elasticities are labour,  $\alpha = 0.3$ , and capital,  $\beta = 0.9$ . Degree of returns to scale = 1.2.
  - Value data calculated as before and mark-up  $\mu$  is again 0.33.
  - Factor shares are labour,  $a = 0.75$ , and capital,  $(1-a) = 0.25$ .
-



have the same technology, produce the same output and the firm is the unit of observation. This is an important point. If we need to estimate a production function using outputs and inputs summed over different firms, we encounter all the well-known aggregation problems. As monetary data have to be used in estimating this aggregate production function, we can explain why regressions using these data give a good fit to the aggregate data when theoretically they should not (Fisher, 1971b; Felipe & McCombie, 2005). However, the problem is even more fundamental than this. As will be demonstrated, the accounting identity presents insurmountable problems of interpretation, even when there are no aggregation problems of any kind regarding functional forms, or affecting output, labour or capital (see Felipe & Fisher, 2003) or problems of the type discussed in the context of the Cambridge debates regarding the nature and construction of capital stocks (Cohen & Harcourt, 2003).

The important aspect of our simulations is that they show how the use of monetary data can give results at variance with the true magnitudes of the underlying production functions and, therefore, misleading numerical estimates of both the parameters of the production function and of the 'rate of technical progress'. For clarity, we will confine the term 'technical progress' to that calculated using physical data; and the term 'total factor productivity growth' to that calculated using monetary data.

#### *Cross-Firm Estimation of the Production Function*

In the first example, data in physical units were generated for 10 firms for one period under the assumption that they all have identical Cobb–Douglas constant-returns-to-scale production functions given by:

$$Q_i = A_0 L_i^\alpha K_i^{(1-\alpha)} \quad (6)$$

where  $Q_i$  is the number of units of homogeneous output, generated as a random variable;  $K_i$  is the number of identical machines which are specific to the particular industry, also generated as a random variable;  $L_i$  is the level of labour input, generated through the production function;  $A_0$  takes the same value for all firms and was normalized to unity. The parameters  $\alpha$  and  $(1 - \alpha)$  are the output elasticities of labour and capital, respectively, and are constructed to take values of 0.25 and 0.75.<sup>9</sup> The output elasticities were thus deliberately chosen to be the converse of the factor shares found in the NIPA.

In order to generate the monetary values, each firm sets prices as a mark-up on unit labour costs, i.e.

$$p_i = (1 + \mu)w_i L_i / Q_i \quad (7)$$

The mark-up  $\mu$  is the same for all firms and takes a value of one third, so  $(1 + \mu) = 1.333$ . The wage rate is the same across firms, as is the profit rate  $r$ , which takes a value of 0.10. The monetary value of the capital stock was calculated residually through the accounting identity, equation (1), as  $J_i \equiv (V_i - wL_i)/r$ , where  $V_i$  is value added, constructed as  $V_i = p_i Q_i$  for each firm using equation (7). The values of the factor shares are directly calculated using these monetary data. Labour's share is calculated as  $a_i = (wL_i)/V_i$  and capital's share as  $(1 - a_i) = (rJ_i)/V_i$ . It should also be noted that  $a_i = 1/(1 + \mu)$ , and so it takes a value of 0.75 for each firm, with a small

variation owing to the error term added. The mean value of labour's share for the 10 firms is 0.744 (with a range of 0.698–0.795).

The mean value of the capital–output ratios in monetary terms ( $J_i/V_i$ ) is 2.57 with a range of 2.24–3.18. These values are very close to what are observed empirically, and are the result of a roughly constant rate of profit and a constant factor share of capital. As  $J/V = (1 - a)/r$ , where  $(1 - a)$  is capital's share and, as noted above, is approximately equal to 0.25 and the profit rate is 0.10, the capital–output ratio will not differ much from 2.50. As we are dealing with individual firms and we design the simulations, we know both the physical data and the monetary values, as we know the prices. However, let us assume that the prices are unknown to the investigator, as is usually the case, because the output and capital stocks for different firms are aggregated in the NIPA, etc., using monetary values. Consequently,  $V$  and  $J$  (in constant prices, although because we only have one period, the distinction between current and constant prices does not arise) were taken as proxies for  $Q$  and  $K$ .

These monetary data were then used to estimate a cross-firm production function. The results of the estimation are:

$$\ln V = 2.867 + 0.750 \ln L + 0.250 \ln J \quad \bar{R}^2 = 0.999$$

$$(478.77) \quad (136.40) \quad (45.41) \quad \text{s.e.r.} = 0.0025$$

This gives a remarkably close fit to the Cobb–Douglas production function, which is to be expected given the method used to construct the data. However, some of Douglas and his colleagues' early studies, which used real, as opposed to simulated, cross-sectional (industry or region) data, also found very close statistical fits.<sup>10</sup> The sum of the estimated coefficients is 1.00 and this is not significantly different from unity (the value of the  $t$ -ratio testing this hypothesis is 0.02). With the close correspondence between the supposed 'output elasticities' and factor shares calculated from the data (0.750; 0.744 and 0.250; 0.256), it is little wonder that such results could be interpreted as providing evidence in favour of competitive markets and disproving the Marxian argument, as Douglas (1976, p. 914) claimed (See McCombie, 1998b and Felipe & Adams, 2005).

This is notwithstanding the fact that factors are *not* paid their marginal products in physical terms in our simulation data. Competition could force firms to be  $x$ -efficient so that firms do hire the factors of production up to the point where their physical returns equal their factor rewards in terms of the commodity produced. This would determine, *inter alia*, the optimal  $L_i/Q$ , which is used in the mark-up pricing equation. However, using monetary data would still give estimates of the 'output elasticities' equal to  $1/(1 + \mu)$  and  $\mu/(1 + \mu)$ , respectively.

It should be emphasized that the estimated 'output elasticities' are, of course, not the same as the 'true' output elasticities of the micro-production function. In other words, the true output elasticity of labour is 0.25, but the estimate using monetary data is 0.75.

The goodness of fit is dependent upon the degree of variation in the mark-up. With identical mark-ups, the fit is exact (and estimation is not possible because of perfect multicollinearity). Indeed, it is the constant mark-up that is solely responsible for generating the spurious Cobb–Douglas relationship. To demonstrate this, the physical values of the three series  $Q$ ,  $L$ , and  $K$  were next generated as random numbers.  $V$  and  $J$  were calculated as before. The estimation yielded a very good fit to the Cobb–Douglas with the values of the 'output elasticities' the same as before

(the result is not reported here). This does not necessarily mean that we are postulating that that output is actually a random function of the inputs. However, when one considers the complex production processes of any modern firm, there may be some individual parts of the process subject to fixed coefficients, whereas others are subject to differing elasticities of substitution, to say nothing of differences between plants in managerial and technical efficiencies. Thus, the randomness may simply be a reflection of the severe misspecification error inherent in specifying the micro-production function as a Cobb–Douglas. However, the important point to note is that even in this case, where there is no well defined micro-production function, the use of value-added data will give the impression that there exists a well-behaved aggregate Cobb–Douglas production function.

#### *Rate of Technological Progress and Total Factor Productivity Growth*

In order to calculate the growth of total factor productivity, we need the growth rates of output, capital and labour. We assumed that outputs of the 10 hypothetical firms grow at different rates (we only have one growth period), but, for expositional purposes only, the series were constructed such that the growth rate of the physical capital–output ratio is zero (i.e. output and capital grow at the same rate) for all firms. It was also assumed that each firm experiences the same rate of technical progress ( $\varphi_i$ ), namely, 0.5% per annum, which is equal to  $\varphi_i = \alpha(\hat{Q}_i - \hat{L}_i)$ . This is due to the fact that the underlying production functions are Cobb–Douglas and that the growth of output equals the growth rate of capital, where both are measured in physical terms. Hence, the growth rate of employment for each firm is given by as  $\hat{L}_i = \hat{Q}_i - (\varphi_i / \alpha)$ , where  $\varphi_i = 0.5\%$ , as noted above. The output elasticities of labour and capital in *physical* terms are again 0.25 and 0.75. However, the average factor shares in *monetary* terms are 0.745 (with a range from 0.698 to 0.795) and 0.255, and the aggregate shares are also 0.745 and 0.255, as each firm has the same mark-up. This means that the labour share of each firm is the same and if we aggregate over firms, the aggregate share comes to about the same value.

Although the investigator cannot use the values of the variables in physical terms, we are able to do so simply because we have constructed the data set. Consequently, we can calculate the rate of technical progress for each hypothetical firm separately using the standard growth accounting equation; that is,

$$\varphi_i \equiv \hat{Q}_i - a\hat{L}_i - (1-a)\hat{K}_i \quad (8)$$

where  $a$  and  $(1-a)$ , the factor shares of labour and capital, are assumed to equal the output elasticities  $\alpha$  and  $(1-\alpha)$ , 0.25 and 0.75, respectively.

As the rate of technical progress is the same for each firm, we can talk about the rate of technical progress being 0.5% per annum, even in the case where we assume that the physical outputs of the various firms are not homogeneous.

However, the individual prices of the various firms are not available to the investigator and so it is not possible to extract data on the physical units of output. All that can be used in empirical work, as is usually true in practice, is the constant price value of output and of the capital stock. The growth of total factor productivity in these circumstances is given by:

$$TFPG_i \equiv \hat{V}_i - a\hat{L}_i - (1-a)\hat{J}_i \quad (9)$$

where now the shares of labour and capital are 0.75 and 0.25, respectively.

The unweighted mean rate of total factor productivity growth of the individual firms is 1.49% per annum, which, not surprisingly, is almost identical to the rate of total factor productivity growth (1.48% per annum) obtained by aggregating the monetary data over all 10 firms and using these in an aggregate version of equation (9).

Thus, the use of physical data yields technical progress accounting, on average (the unweighted mean), for  $\varphi_i = 0.25(\hat{Q}_i - \hat{L}_i)$ , that is, 25% of labour productivity growth, with a very small difference between firms owing to the small random element introduced for the reasons noted above.<sup>11</sup> However, the use of monetary data for each of the 10 firms gives a mean value of the rate of total factor productivity that is  $TFPG_i = 0.75(\hat{V}_i - \hat{L}_i)$ , or 75% the growth of labour productivity, with a range from 80% to 70%. The figure using the aggregate data (i.e. using the aggregate values of output, labour and capital) is 74%. The reason for the marked difference between these values and the 'true' rate of technical progress is that labour's share of output in value terms is 0.75, while the 'true' output elasticity of labour is 0.25.

It is worth noting that  $TFPG$  in equations (3a) and (3b) can be written as  $TFPG_t \equiv \hat{V}_t - a_t \hat{L}_t - (1 - a_t) \hat{J}_t$  which equals  $a_t(\hat{V} - \hat{L}) + (1 - a_t)(\hat{V} - \hat{J})$ . The last expression is a weighted average of the growth rates of labour and capital productivity. Therefore, it could be argued that  $TFPG$  is an average measure of productivity growth. This interpretation faces, however, the problems discussed in this subsection, namely, that the figure computed is not equivalent to the true rate of technical progress. The growth of labour and capital productivity in monetary terms are unlikely to be the same as those measured in physical terms and the weights are arbitrary, in that the shares will change because of changes in, for example, the mark-up.

Consequently, the use of monetary data produces a significantly different estimate of the rate of technical progress, compared with the true value obtained using physical data. Even with well-defined underlying Cobb–Douglas micro-production functions expressed in physical terms, the use of monetary data as a proxy for output can give very misleading estimates of the rate of 'technical progress'.

#### *Increasing Returns to Scale and Total Factor Productivity Growth*

What happens if the individual firms are subject to increasing returns to scale when physical data are used? To examine this question, we first estimated the cross-firm production functions using monetary data when the micro-production functions exhibit the same degree of increasing returns to scale. The data for the inputs in physical terms were the same as those used in the previous simulation, with the exception that now the elasticities were multiplied by 1.20, so  $\alpha' = 0.30$  and  $\beta' = 0.90$ . This represents a substantial degree of returns to scale and results in a value of output that is significantly larger than when constant returns to scale are imposed. The monetary data were calculated the same way as before, with a mark-up once again of one-third.

Estimating the unrestricted Cobb–Douglas production function gives a result that is virtually identical to that for constant returns to scale, and reported above,

except for a change in the value of the intercept. Consequently, we do not report the results here. The estimates of the putative output elasticities are once again very close to the observed monetary values of the factor shares and sum to unity, thereby erroneously suggesting that the production process of the various firms are subject to constant returns to scale. The reason for this seemingly paradoxical result is that the calculation of value added is given by  $pQ = V = (1 + \mu)wL$  and as nominal wages and the level of employment are the same as before, so is the constant price measure of value added, although the price per unit is now lower. Recall that we showed above that with monetary data estimation of the production function would automatically yield estimates of the 'output elasticities' equal to those of the factor shares.

Next, we calculated the rate of technical progress and the growth of total factor productivity. For comparability with the constant returns to scale case, the growth rates of physical output, capital and labour were the same as before. The rate of technical progress was calculated using the physical data as

$$\varphi_i = \hat{Q}_i - \alpha' \hat{L}_i - \beta' \hat{K}_i \quad (10)$$

where  $\alpha' = 1.2\alpha$  and  $\beta' = 1.2(1 - \alpha)$ . It can be seen that the rate of technical progress calculated using equation (10) will differ from the 0.5% in the case of constant returns to scale. In fact, it will be on average lower, given the larger weights on the growth of the factor inputs. The rate of technical progress, calculated using equation (10) for each firm now varies considerably across firms (for reasons of space we do not report the full results).<sup>12</sup> The unweighted mean is (coincidentally) 0.00% per annum, with a range of  $\pm 0.4$  percentage points per annum.<sup>13</sup>

However, the growth rates of total factor productivity of the individual industries, calculated using equation (9) and monetary data, are again all approximately 1.5% per annum. This is because the shares of labour and capital in monetary terms are once again 0.75 and 0.25, and the growth rates of  $V$ ,  $L$  and  $J$  are the same as before.<sup>14</sup> Thus, the use of monetary data can give a very misleading estimate of the true rate of technical progress. The use of monetary data erroneously ascribes the effect of increasing returns on increasing the efficiency of the factors of production to the rate of technical progress.

### Summary

The actual figures in the simulations are, of course, arbitrary as they depend on the assumed rates of growth of physical output, capital and employment. Nevertheless, we can draw some conclusions from these simulations.

- The growth of total factor productivity depends crucially on the weights attached to the growth of capital and labour. The growth accounting approach assumes that factors are paid their marginal products and hence the technologically determined output elasticities will equal the factor shares. However, when monetary data are used we have shown that the factor shares will *always* equal the putative output elasticities and both are determined by  $1/(1 + \mu)$  (for labour) and  $\mu/(1 + \mu)$  (for capital), where  $1 + \mu$ , it will be recalled, is the mark-up. The estimates of the output elasticities using monetary data will almost certainly differ from the true ones (always assuming that there is a well-defined micro-production function in physical terms in the first place).

- Where it is possible to compare the 'true' growth rate of technical progress with the growth of total factor productivity in monetary terms (namely at the firm level here), the two values will probably differ markedly. In general, it is not possible to recover the physical quantities (of both output and capital) from the monetary data through the use of individual prices and so resort is made to the use of monetary data in real terms with potentially very misleading results. Where the physical data is available, it can only be at a very low level of aggregation. It requires each output and capital good to be measured separately in physical terms.
- The problems posed by the accounting identity are independent from, and in a sense more fundamental than, either the aggregation problem or the Cambridge Capital Theory Controversies. This is because the problems arise even when the last two do not pose any problems.

It should be clear by now that from the simulations that all estimations of aggregate production functions do is to approximate the NIPA accounting identity, and as such they are exercises that, although algebraically correct, stand without any theoretical foundation.

What then are we to make of Barro's (1999, p. 123) argument that an economy that experiences an increase in both its real wage and profit rates must have increased its overall level of productivity? It could be argued that  $TFPG_t = a_t \hat{w}_t + (1 - a_t) \hat{r}_t$  measures such a rate of growth of efficiency. Certainly, under these circumstances one can say that the economy is better off, because obviously this measure contributes positively to output growth. The point to note is that, as we mentioned above, it is not possible to ascribe this unambiguously to the result of technical change the way it is done in the neoclassical model (i.e. by claiming that this concept is derived from a testable model). There is no reason to assume that factor shares, i.e. the appropriate or theoretically justified weights according to Barro (1999, p. 123), equal the output elasticities of the true aggregate production function (if it, in fact, exists), or that production is necessarily subject to constant returns to scale (although this is what the use of monetary data will show). The derivation is simply a tautology resulting from an identity with no behavioural assumptions or implications, except that factor shares in monetary terms are constant.<sup>15</sup>

Suppose, for the sake of the argument, that the factor shares are determined by the relative bargaining power of labour *vis-à-vis* capital. Let us assume further that capital's share increases and labour's share falls as a result of changes in legislation relating to trade unions. Even though in physical terms the growth rates of output and inputs, and hence of technical change, remain the same by assumption, the calculation of *TFPG* will show a decline compared with its previous value. Hence, the concept of total factor productivity growth cannot be given a technological interpretation.

## Conclusions

This paper has shown how the use of monetary data may give very misleading estimates of the true rate of technical progress. The simulation exercises have shown how the use of monetary data will give erroneous estimates of the sources of growth. For example, even though the micro-production functions each exhibited increasing returns to scale with labour and capital's output elasticities taking a value of 0.3 and 0.9, respectively, empirical estimation with monetary data,

however, gives the traditional values of 0.75 and 0.25. Moreover, the estimates of *TFPG* are very different from the true values at the industry level.

How are then we to interpret the Solow residual, in either its primal or dual form? For it to be seen as a uniquely determined measure of technical progress requires the assumption that the relationship between the inputs and output of the economy (or individual industries) can be represented by an aggregate production function. This seems implausible, given the theoretical objections that have been levelled against it. As Hahn (1972, cited by Blaug, 1974, p. 19) put it:

It has often been the case that a neo-classical theory has been attempted in terms of aggregate production functions and aggregates like capital. Except under absurdly unrealistic assumptions such an aggregate theory cannot be shown to follow from the proper theory and in general is therefore open to severe logical objection. ... It is extremely unfortunate, and to some extent the fault of its practitioners, that neo-classical theory has come to be identified with this aggregation theory. On purely theoretical grounds there is nothing to be said in its favor. The view that that nonetheless it 'may work in practice' sounds a little bogus and in any case the onus of proof is on those who maintain this.

Recourse to empirical estimation does not shed any light on the issue, and, indeed, it is always possible to get a perfect fit to the data in a form that resembles an aggregate production function.

Barro (1999) has suggested how the growth accounting approach may be adjusted to incorporate the arguments of the endogenous growth models. However, this approach also requires the existence of a well-behaved aggregate production function. The problem is that physical data for industrial processes are not readily available and where it is, it is at a very low level of aggregation. Moreover, it is very unlikely that the simple (if not simplistic) Cobb–Douglas, or even the more flexible functional forms such as the translog, are likely adequately to model the production processes of a modern manufacturing plant or a firm in the service sector.

Solow once remarked that: 'I have never thought of the macroeconomic production function as a rigorous justifiable concept. In my mind, it is either an illuminating parable, or else a mere device for handling data, to be used so long as it gives good empirical results, and to be abandoned as soon as it does not, or as soon as something better comes along' (Solow 1966, pp. 1259–1260). Perhaps such a time has arrived. Felipe & Fisher (2003, p. 257) concluded their survey on the aggregation literature as follows: 'Macroeconomists should pause before continuing to do applied work with no sound foundation and dedicate some time to studying other approaches to value, distribution, employment, growth, technical progress, etc., in order to understand which questions can legitimately be posed to the empirical aggregate data.' The problems associated with the interpretation of neoclassical growth accounting exercise merely serves to reinforce these conclusions.

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## Notes

1. The national accounts only give data for total wages ( $W$ ) and total profits ( $\Pi$ ) but employment and capital stocks (calculated using the perpetual inventory method) are readily available from many statistical sources such as the OECD and the US Bureau of Labour Statistics.
2. This criticism, in a very sketchy form, can be traced back to the Reder (1943). More recent expositions include Phelps Brown (1957), Shaikh (1980) and Simon (1979).
3. The notation in Barro's quotations has been changed to make it consistent with that in this paper.
4. Alternatively, the values of one or more of the variables must be adjusted to ensure that the identity holds.
5. Hart suggests that this may possibly be due to a reorganization of labour agreed to by the unions. To the extent that this leads to increased efficiency and output, then the causation could be said to run from right to left.
6. Reder (1943, p. 260) is an exception: 'in economic theory a production function relates *physical* quantities of output to quantities of *physical* input of the factors. But the Cobb-Douglas function ... relates the *value* added in manufacturing, defined as product, to the physical input of labor services (labor) and the value of plant and equipment owned (capital)' (italics in the original).
7. As a consequence, parameters such as the aggregate elasticity of substitution are meaningless. In the words of Fisher *et al.* (1977): 'the elasticity of substitution in these production functions is an "estimate" of nothing; there is no true aggregate parameter to which it corresponds' (Fisher *et al.*, 1977, p. 312).
8. These results extend an earlier simulation based on time-series data (McCombie, 2001).
9. To prevent perfect multicollinearity, a small random variable was added to these and, where necessary, other variables used in the simulation.
10. For example, Douglas (1976, p. 906) reports the following results for a production function based on American cross-section studies, 1904, 1909, 1914 and 1919.

Year	$\alpha$	$\beta$	$\alpha + \beta$
1904	0.65 (32.5)	0.31 (15.5)	0.96
1909	0.63 (31.5)	0.34 (17.0)	0.97
1914	0.61 (30.5)	0.37 (18.5)	0.98
1919	0.76 (38.0)	0.25 (12.5)	1.01
Average	0.66 (33.0)	0.32 (16.0)	0.98

Notes:  $t$ -values in parentheses. Total number of observations: 1490.

11. This is because there is no growth in the physical capital-output ratio,  $\varphi_i = \alpha(\hat{Q}_i - \hat{L}_i)$  and  $\alpha$ , the physical output elasticity of labour, is equal to 0.25. Hence the rate of technical progress equals one quarter of the growth of labour productivity.
12. This is because while the growth rates of  $Q$  and  $K$  are the same between firms, employment growth rates differ and so the change in weighting causes the rate of technical change now to differ across firms.
13. As we cannot sum across the physical quantities, we cannot calculate a meaningful average rate of technical progress, as the individual rates cannot be unambiguously or uniquely weighted. Nevertheless, we did calculate the unweighted mean.
14. With a constant mark-up of 1.333, the shares will be always 0.75 and 0.25, regardless of the technical conditions of production (e.g. the degree of returns to scale).
15. We have adopted a mark-up pricing policy to generate constant shares in value terms. However, there are any number ways that this could occur (e.g. through a bargaining model or a Kaldorian model of distribution).

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