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The CES Production Function, the accounting identity, and Occam's razor

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This paper reconsiders the argument that empirical estimations of aggregate production functions may be interpreted merely as statistical artefact. The reason is that Occam's razor, or Herbert Simon's principle of parsimony, suggests that the aggregate production function, together with the side equations derived from the usual neoclassical optimizing conditions, simply reflect the underlying accounting identity that value added definitionally equals the wage bill plus total profits. This argument is illustrated with respect to the empirical evidence presented by Arrow, Chenery, Minhas and Solow (*Review of Economics and Statistics*, **XLIII**, 225–50, 1961) and which led them to derive the Constant Elasticity of Substitution aggregate production function. It is shown that their results are more parsimoniously explained with reference to the underlying accounting identity than to any technological relationship.

I. INTRODUCTION[1]

> In testing theories aimed at explaining empirical phenomena, it is not enough to satisfy ourselves that the observed data are consistent with the theory. We must also ask whether the data can be explained equally well by other, perhaps weaker and simpler, theories. (Simon, 1979a)

It has long been appreciated that the theoretical foundations of the aggregate production function are tenuous in the extreme. As Fisher (1971, p. 305) commented: 'Recent work [on aggregation] has shown pretty clearly that the conditions under which the production possibilities of a technology diverse economy can be represented by an aggregate production function are far too stringent to be believable.' But there is a puzzle. As he continues: 'Yet aggregate production functions apparently work nevertheless and do so in a way which is *prima facie* not easy to explain.' A classic example of this is the paper by Arrow, Chenery, Minhas, and Solow (1961) (hereafter ACMS) which first popularized the Constant Elasticity of Substitution (CES) production function. In the course of their paper ACMS estimated a side relationship derivable from the CES production function, namely the regression of the logarithm of labour productivity on that of the wage rate. This relationship was estimated using cross-country data for a number of countries for selected industries at the three-digit SIC level. In spite of the wide disparities in the level of economic development of the countries in the sample, the R^2s were uniformly high and the regression coefficients well determined.

But, *pace* Fisher, there has been an explanation of the good statistical fit that the production function usually gives which dates back to Phelps Brown (1957).[2] This is the proposition that all that the data are actually reflecting is an underlying accounting identity and, consequently, it is not surprising that the coefficients are usually well deter-

[1] Professors Fisher, Fuchs, and Solow provided useful comments on a previous version. This should not be taken that they necessarily agree with the arguments put forward here.

[2] The antecedents of this critique may be found in Marshak and Andrews (1944).

mined and the correlation coefficient not far from unity.[3] Phelps Brown's arguments relating to cross-industry estimations were later formalized by Simon and Levy (1963) in a brief note. The problems that the identity poses for the time series estimations of the Cobb–Douglas production function were demonstrated by Shaikh (1974), and putatively refuted by Solow (1974). (But see Shaikh, 1980, and McCombie, 1998, 2000.) Simon (1979a) drew together some of these criticisms, and he briefly touched on the CES production function and the ACMS study (Simon, 1979a, p. 469). Nevertheless, in spite of the fact that he thought that these criticisms were of sufficient importance to be mentioned in his Nobel lecture of the same year (Simon, 1979b, p. 497), they have been almost entirely ignored in the literature.

The purpose of this paper is to revisit the ACMS paper and present some supplementary evidence in support of Simon. The rest of the paper is organized as follows. First, Section II discusses the argument that while the data may be reflecting an aggregate production function, Occam's razor, or the principle of parsimony as Simon put it, suggests that nothing more is being captured than the accounting definition of value added. In Section III, the CES production function is discussed in the light of the principle of parsimony, and the cross-section regression results of ACMS are reconsidered. While the points considered here could have been made with more recent data, the importance of the ACMS paper in the development of production theory makes their data set a fitting choice. Moreover, the present arguments may have some importance for the history of economic thought. In Section IV the time-series results of ACMS are discussed using Solow's (1957) data. Section V concludes the paper and summarizes the main findings.

II. THE AGGREGATE PRODUCTION FUNCTION AND THE ACCOUNTING IDENTITY

In this section the argument that all that estimations of production functions are doing is capturing an underlying accounting identity (albeit sometimes misspecified) will be briefly considered. Hence, regressing output on the inputs is bound, almost by definition, to give a very good statistical fit. Such estimations, because of the underlying identity, can provide no independent test of the hypothesis of whether or not an aggregate production function exists or, even if it does, what the true structural parameters are.[4] There are a number of ways this criticism can be made; but to begin with Simon and Levy's (1963) argument for the Cobb–Douglas will be followed due to its simplicity. Later on this will be extended. Here the concern is with cross-industry data or statistics for the same industry but using different spatial observations (e.g. regions or countries). It is assumed that the wage and the rate of profit do not vary greatly between the different industries or spatially within the same industry. (For the sake of argument, it will henceforth be assumed that different industries are being dealt with.)

The value added of the ith industry is defined as:

$$Q_i \equiv wL_i + rK_i \tag{1}$$

where Q is output (value added), w is the real wage, r is the average rate of profit, L is employment and K is the capital stock. Labour's share is defined as $a_i \equiv wL_i/Q_i$.

The Cobb–Douglas production function is given by:

$$Q_i = AL_i^{\alpha}K_i^{\beta} \tag{2}$$

where α and β are the output elasticities with respect to labour and capital. If Equation 2 is expanded as a Taylor series around the mean value of the variables, i.e. around $\bar{Q}$, $\bar{L}$, and $\bar{K}$, and terms to the power of two or greater are ignored, the following approximation is obtained:

$$Q_i \approx \bar{Q} + \alpha(\bar{Q}/\bar{L})(L_i - \bar{L}) + \beta(\bar{Q}/\bar{K})(K_i - \bar{K}) \tag{3}$$

Expanding the accounting identity (Equation 1) yields

$$Q_i = \bar{Q} + w(L_i - \bar{L}) + r(K_i - \bar{K}) \tag{4}$$

Comparing expressions (3) and (4) leads to the conclusion that $w = \alpha(\bar{Q}/\bar{L})$ (or $\alpha = w\bar{L}/\bar{Q}$), and $r = \beta(\bar{Q}/\bar{K})$ (or $\beta = r\bar{K}/\bar{Q}$). Note that this implies that $\alpha + \beta = 1$.[5] Therefore, if, in fitting a Cobb–Douglas function, one finds a value of the elasticity α in agreement with the actual labour share a (and that β equals capital's share $(1 - a)$), this cannot be taken to imply that the underlying production function is truly a Cobb–Douglas. But the important aspect to stress is that the Cobb–Douglas production func-

[3] Fisher (1971, p. 305, footnote 3) was aware of Phelps Brown's article but disregarded it on the grounds that Phelps Brown 'dismisses the time series results as poor or implausible, largely because of their failure to allow for technical change' and that his arguments 'do not show why a cross-sectionally estimated production function should give reasonable wage predictions for years far from that of the original cross section'. Turning to the first point, Fisher is only partially correct. It is true that the original Cobb and Douglas's time series estimations which Phelps Brown criticized did not make any allowance for technical change. But Phelps Brown was well aware of other studies which *did* include a time trend. These were dismissed by Phelps Brown (1957, p. 350) because the 'results have not been acceptable' (i.e. the estimated coefficients differ markedly from the factor shares and that of capital was often statistically insignificant). Why this occurs notwithstanding that an identity if being estimated is discussed in the text below. It is not clear to us exactly what Fisher means in his dismissal of the Phelps Brown's criticism of the cross-industry results.

[4] This is true even if no explicit recourse is made to the marginal productivity theory of factor pricing in deriving the estimating equation.

[5] This is because $\alpha + \beta = (w\bar{L} + r\bar{K})/\bar{Q} = 1$.

tion will, should the real wage and profit rates be roughly constant, provide a close approximation to the accounting identity. This will be true no matter what the actual underlying aggregate technology is, or even when, because of aggregation problems, there is no such thing as a well-behaved aggregate production function that reflects the technology of the economy or industry. The approximation will not necessarily be exact, of course, because in the accounting identity the factor shares are not necessarily constant, whereas they are in the Cobb–Douglas function. Consider the case where the wage and rate of profit are constant. Simon (1979a) shows that if a wide range in the capital–labour ratios is taken that is much larger than occurs in reality, the error obtained from mistakenly fitting the Cobb–Douglas function is generally small; the predicted value differs from the observed by less than 10%. 'Since in the data observed, most of the sample points lie relatively close to the mean value of [the labour/capital ratio], we can expect *average* estimating errors of less than 5 per cent' (Simon, 1979a, p. 466).[6]

The argument may be generalized to other putative production functions. To see this, assume that there is a continuum of firms. Recall that the identity is given by $Q_i \equiv w_i L_i + r_i K_i$. Taking the total differential of the identity and expressing it as proportionate rates of change, one obtains:

$$dQ_i/Q_i \equiv a_i(dw_i/w_i) + (1-a_i)(dr_i/r_i) + a_i(dL_i/L_i) + (1-a_i)(dK_i/K_i) \quad (5)$$

where $a_i \equiv w_i L_i/Q_i$ and $(1-a_i) \equiv r_i K_i/Q_i$. The cross-sectional production function may be written generally as $Q_i = Af(L_i, K_i)$ where A is a constant. Differentiating this with respect to time and expressing as proportionate changes gives:

$$dQ_i/Q_i \equiv \alpha_i(dL_i/L_i) + \beta_i(dK_i/K_i) \quad (6)$$

If $a_i(dw_i/w_i) + (1-a_i)(dr_i/r_i)$ is zero, it can be seen from a comparison of Equations 5 and 6 that α_i must equal a_i and β_i must equal $(1-a_i)$, even though no assumption has been made about the state of competition or that factors are paid their marginal products. Production function studies estimate specific functional forms of Equation 6, including the CES as will be seen below. But this does not obviate problem that all that is being estimated is an approximation to the identity.

For example, assume that factor shares are constant for reasons that have nothing to do with the form of the production function. The identity, Equation 5, may be integrated to give the specific functional form, $Q_i \equiv Bw_i^a r_i^{(1-a)} L_i^a K_i^{(1-a)}$, which is identical to the Cobb–Douglas 'production function', namely, $Q_i = AL_i^\alpha K_i^\beta$, provided that $w_i^a r_i^{(1-a)}$ is roughly constant, or is orthogonal to $L_i^a K_i^{(1-a)}$. It follows that $\alpha \equiv a$ and $\beta \equiv (1-\alpha)$.

In this last case, the accounting identity incorporates the stylized facts of constant factor shares, but there are a number of explanations that are independent of the production function that can account for this, such as a constant mark-up on unit costs or the Kaldorian macroeconomic theory of distribution. However, since a Cobb–Douglas production function also generates constant shares, it is not possible to reject the hypothesis that this is the true underlying production function. Nevertheless, the work on aggregation problems, as noted in the introduction, most notably by Fisher (see, for example, Fisher, 1969), has shown that the conditions for the existence of an aggregate production function are so stringent as to rule it out on theoretical grounds. Fisher (1987) summarized the position as 'that the analytic use of such aggregates as "capital", "output", "labour" or "investment" as though the production side of the economy could be treated as a single firm is without sound foundation', although he added that 'this has not discouraged macroeconomists from continuing to work in such terms'.[7] Next a consideration of how the CES 'production function' may also be interpreted as simply reflecting the underlying identity will be explored.

III. THE CES PRODUCTION FUNCTION AND THE ACCOUNTING IDENTITY

Since the publication of the seminal paper by ACMS, the CES production function has been widely used in both theoretical and empirical work. The starting point of ACMS's work was the empirical observation that the value added per unit of labour used within a given industry varies across countries with the wage rate. Consequently, ACMS fitted a regression of the logarithm of labour productivity on a constant and the logarithm of the real wage rate, that is (Equation (Ib) in ACMS),

$$\ln(Q_i/L_i) = c_1 + b_1 \ln w_i \quad (7)$$

The data used were cross-sections of 19 countries for 24 three-digit SIC manufacturing industries for the early 1950s (not the same year for every country). The results

[6] Not surprisingly the same argument applies to time series data. This is discussed in Section III and has been analysed in Shaikh (1974, 1980), McCombie (2000), McCombie and Dixon (1991) and Felipe (2000).

[7] It might look as if the problem faced is one of econometric *identification* between the production function and the identity. This is not, however, the case. The difficulty is rather one of *interpretation*. It is not an identification problem because the accounting identity is not a behavioural equation and so there are no other variables that one could include in it, even as a matter of principle, to identify the production function.

indicated a highly significant correlation. The authors concluded that '... in 20 out of 24 industries, over 85 per cent of the variation in labour productivity is explained by variation in wage rates alone' (ACMS, p. 228). Since ACMS interpreted the parameter b_1 as the elasticity of output per worker with respect to the wage rate, the authors tested the null hypothesis $H_0 : \hat{b}_1 = 1$. Of the 24 industries, 23 had estimated elasticities below unity, and this was statistically significantly in 14 cases (Table 2 in their paper). This led ACMS to conclude that '... our empirical results imply that elasticities of substitution tend to be less than one, which contrasts strongly with the Cobb–Douglas view of the world' (ACMS, p. 230). This provided the rationale for their derivation of the CES function, together with several auxiliary linear side relations to facilitate its estimation as nonlinear estimation techniques were in their infancy in the early 1960s. The observations of labour productivity were treated as if they came from a constant returns to scale (meta) production function. Postulating perfectly competitive markets, ACMS derived the following equation, under the assumption that the production function exhibits constant returns to scale and a constant elasticity of substitution (Equation 25 in ACMS):

$$\ln(Q_i/L_i) = \ln[(\gamma^{1-\sigma})(\delta)^{-\sigma}] + \sigma \ln w_i \qquad (8)$$

where γ is an efficiency parameter, δ is a distribution parameter, and σ is the elasticity of substitution, and all are constants. Equation 8 provides the theoretical rationale for Equation 7 and ACMS show that the production function underlying Equation 8 is the now well-known CES, namely, $Q_i = \gamma[\delta L_i^{-\rho} + (1-\delta)K_i^{-\rho}]^{-1/\rho}$.

However, the empirical results obtained by ACMS were challenged by Fuchs (1963) who demonstrated that the data used by ACMS did not support their conclusions. 'Under a more reasonable interpretation, the data are substantially consistent with the Cobb–Douglas assumption of an elasticity of substitution of unity' (Fuchs 1963, pp. 436–7). The basis for Fuchs's refutation of ACMS's conclusions was that the nations used by the latter were very diverse (9 developed and 10 developing countries ranging in level of economic development from the United States and Canada to India and Iraq). Fuchs questioned the assumption that both the developed and the less developed countries would have the same parameters, notably the same level of technical efficiency. To test this, he included a dummy variable to differentiate the two groups. The regression results now had only two industries with significant elasticities below one, and another two industries had elasticities above one.[8] Ironically, these results should have vitiated the empirical justification to search for a more general production function compared with the Cobb–Douglas. Fuchs concluded: 'I do not argue that the Cobb–Douglas assumption of unity is correct, but only that the data presented by Arrow *et al.* do not constitute an adequate refutation of it' (Fuchs 1963, p. 438). There is, however, a problem with the interpretation of the regression results when the dummy variable is included. It turns out that the value of the intercept of the less developed countries is larger than that of the advanced countries. If Equation 8 were the correct rationale for the regression, it can be seen that this implies that the level of efficiency of the advanced countries was actually lower than that of the less developed countries, provided b_1 was less than unity.[9]

As stated above, a first purpose of this paper is to suggest that both ACMS and Fuchs (1963) overlooked a fundamental problem that renders their conclusions concerning the regression results problematic. While Fuchs was correct in pointing out that ACMS had erroneously concluded that most of their regressions indicated that b_1 was smaller than one, he also overlooked the fact that Equation 7 above suffers from an *interpretation* problem, and that it could be simply the result of an identity. This was Simon's (1979a) important, but unfortunately neglected, contribution. To see this, note that from the definition of the labour share, labour productivity can be written as:

$$Q_i/L_i \equiv w_i/a_i \qquad (9)$$

If it happens that the labour share is 'sufficiently' constant across countries (i.e., $a_i \approx \bar{a}$, where the subscript i denotes the ith country), Equation 9, an identity, will be equivalent to ACMS's Equations 7 and 8 (to see this, take the logarithm of Equation 9). Given the presence of this underlying identity, it is hardly surprising that the R^2 is so high. Thus, one may argue that the use of regression analysis in Equation 7 would simply be a test for the null

[8] Fuchs included in the group of developed countries the USA, Canada, New Zealand, Australia, Denmark, Norway, the UK, Ireland and Puerto Rico; and in the group of developing countries, Colombia, Brazil, Mexico, Argentina, El Salvador, Southern Rhodesia, Iraq, Ceylon, Japan, and India. If Fuchs had used different dummies for each of the 24 industries, grouping the countries with the highest and lowest labour shares in each industry, he would have found that in all cases, without exception, the estimate of b_1 is one. In other words, the reason why he found four cases with an estimate of b_1 different from one is that in those cases the relationship between 1–0 dummy variable and high–low labour share is broken. Table 2 in the text below shows that in eight industries, the country with the highest labour share is a developing country; and in three cases, the country with the lowest share is a developed country.

[9] This was, of course, appreciated that the time. Various suggestions were made to resolve the paradox, including the fact that the wages in the less developed countries were inaccurately measured. If w is subject to measurement error then the slope coefficient will be biased downwards and an appropriate instrument should be used in the estimation. Of course, if the estimate of b_1 was not statistically different from unity, then the intercept is no longer a function of the level of efficiency and the paradox vanishes. By including a dummy variable, Fuchs found that this became the case.

hypothesis that labour's share in the cross-section is constant. Obviously, if by chance $a_i = \bar{a}$, the OLS regression of Equation 7 (formally equivalent to estimating the logarithmic transformation of the identity, Equation 9) will yield the following: $R^2 = 1$, $\hat{b}_1 = 1$, and $\hat{a} = 1/\exp(\hat{c}_1)$. Comparing Equations 8 and 9, one can see that the estimate of σ must be one. In most cases, however, the labour share will not be exactly constant across countries, and thus the R^2 will be less than unity, and the estimate of σ (*viz.* $\hat{b}_1$) will differ from 1. On the one hand, if labour's share does not vary systematically with the wage rate, then $b_1 \approx 1$. If, on the other hand, labour share and wage rate vary systematically, then $\hat{b}_1 > 1$ (if their covariance is negative) or $\hat{b}_1 < 1$ (if their covariance is positive). But these explanations are compatible with the identity (Equation 9). Thus, due to this problem of interpretation, it is not possible unambiguously to accept this regression as reflecting the parameters of an underlying production function, as ACMS did. It is of course possible that this is the case, and thus $\hat{b}_1$ would be an estimate of the aggregate elasticity of substitution σ.[10] However, the data can provide neither an independent test of this hypothesis nor a test of whether or not the underlying technological structure of the economy is a well-defined CES (or any other) production function.

The regressions for the 24 industries using ACMS's data were re-run and the results are shown in Table 1[11]. This is done for the purposes of recovering the implied factor share, $1/\exp(\hat{c}_1)$, and testing the null hypotheses $H_0 : \hat{b}_1 = 1(\chi_1^2)$, and the joint hypotheses $H_0 : 1/\exp(\hat{c}_1) = a_{ave}$ and $b_1 = 1(\chi_2^2)$. Table 2 shows the average share of labour (a_{ave}) for the various industries, the range (a_{max}; a_{min}) with the corresponding country (country a_{max}; country a_{min}), the standard deviation (s) of the shares, and the coefficient of variation (s/a_{ave}). Several features of the results are worth mentioning:

(i) The present results are slightly different from those of ACMS. The conjecture why the estimate of the slope b_1 is slightly different may be due to the improvement in computer precision and any difference may be important for testing purposes. The difference in the intercept is due to the fact that ACMS appear to have used logs to the base 10 rather than natural logs.

(ii) The factor shares, derived from the constant term, appear to be poorly estimated. The implied factor shares in Table 1 are very different from the average factor shares in Table 2, with the exception of industries 332 (Glass) and 342 (Non-ferrous metals).

(iii) The null hypothesis $H_0 : \hat{b}_1 = 1(\chi_1^2)$ cannot be rejected in 14 cases at the 5% confidence level; and if we increase the confidence level to 1%, then it cannot be rejected in 20 industries. (Remember that this is without including a dummy variable.) The reading of these results is, therefore, different from that of ACMS, and the evidence indicates that the 'elasticity of labour productivity with respect to the wage rate' is generally not significantly different from unity.

(iv) The joint hypotheses $H_0 : 1/\exp(\hat{c}_1) = a_{ave}$; $\hat{b}_1 = 1(\chi_2^2)$ cannot be rejected in only three cases (industries 271, 332 and 342) at the 5% confidence level; and in two more cases (industries 331, 334) if the confidence level is increased to 1%.

(v) The information in Table 2 indicates that the dispersion in factor shares is rather large, apart from possible errors in the data. Some of the labour shares in Table 2 are implausible (in particular a_{min}) and throw into doubt the quality of the data. Given this, it is not surprising the overall poor estimates of the factor shares shown in Table 1.

(vi) The fact that almost all point estimates of the slope coefficients reported in Table 1 are less than one ($\hat{b}_1 < 1$) indicates that the size of the labour share is positively correlated with the real wage. This is confirmed in Table 3, which reports the results of estimating $\ln a_i = \mu_1 + \mu_2 \ln w_i$, where the slope parameter μ_2 is simply an estimate of the bias in the labour share identity.[12] How can this relationship be explained other than by invoking an aggregate production function? It could simply be that the mark-up varies with the level of development. The mark-up is higher in the less developed countries which are less unionized and generally have less employment rights. This is of course a behavioural

[10] In this case one could even argue that the data represent *equilibrium* observations, as Professor Solow indicated in private correspondence.

[11] Since the estimation of Equation 7 is merely that of an identity, a number of putative econometric problems are disregarded. For example, it could be argued that the error term in Equation 9 should have an expected value of 1, and be distributed lognormally. Therefore, the error term in Equation 7 would be distributed normally, but not with zero mean. This has implications for the interpretation of the constant (it would be a biased estimate). However since we are depriving the exercise of any behavioural interpretation, the error term here is simply the deviation from the identity.

[12] To see this, note that the identity can be written as $\ln(Q_i/L_i) = d + b^* \ln(w_i) + c^* \ln(1/a_i)$. In this case econometric estimation will yield $d = 0$; $b^* = c^* = 1$. But what is estimated is $\ln(Q_i/L_i) = c_1 + b_1 \ln(w_i)$. This implies that $b_1 = b^* + c^*\{\text{Cov}[\ln(w_i), \ln(1/a_i)]/\text{Var}[\text{in}(w_i)]\}$, or $b_1 = 1 + \text{Cov}[\ln(w_i), \ln(1/a_i)]/\text{Var}[\ln(w_i)]$, since $b^* = c^* = 1$. The ratio of the covariance to the variance is the bias in the estimation of the identity, and it is given by the parameter μ_2 in the auxiliary regression.

Table 1. *ACMS regression: Equation 5*

ISIC No.	Industry	$\hat{c}_1$	$\hat{b}_1$	Implied $\hat{a}$	$\bar{R}^2$	χ_1^2	χ_2^2
202	Dairy products	2.90 (7.59)	0.72 (13.02)	0.05 (2.62)	0.92	25.94	12 356
203	Fruit and vegetable canning	1.86 (3.75)	0.85 (11.50)	0.15 (2.02)	0.91	3.88	143
205	Grain and mill products	1.61 (2.47)	0.91 (9.40)	0.20 (1.53)	0.85	0.88	14.49
206	Bakery products	1.30 (3.26)	0.91 (15.49)	0.27 (2.51)	0.94	2.18	24.25
207	Sugar	2.50 (3.16)	0.78 (6.79)	0.08 (1.26)	0.79	3.64	461
220	Tobacco	2.99 (2.89)	0.75 (4.97)	0.05 (0.96)	0.63	2.65	786
231	Textile-spinning and weaving	2.02 (4.55)	0.80 (12.01)	0.13 (2.25)	0.89	8.44	712
232	Knitting mills	2.10 (4.84)	0.78 (12.24)	0.12 (2.29)	0.91	11.29	2224
250	Lumber and wood	1.61 (3.72)	0.86 (13.11)	0.20 (2.30)	0.91	4.60	133
260	Furniture	1.23 (3.44)	0.89 (17.04)	0.29 (2.78)	0.95	3.89	79
271	Pulp and paper	1.34 (1.89)	0.96 (9.58)	0.26 (1.41)	0.85	0.12	1.31
280	Printing and publishing	1.56 (3.89)	0.86 (15.32)	0.21 (2.49)	0.94	5.36	252
291	Leather finishing	1.66 (3.92)	0.85 (13.81)	0.19 (2.35)	0.92	5.36	192
311	Basic chemicals	2.23 (4.37)	0.83 (11.47)	0.11 (1.96)	0.89	5.55	406
312	Fats and oils	2.30 (3.71)	0.83 (9.31)	0.10 (1.61)	0.87	3.21	151
319	Miscellaneous chemicals	1.84 (4.43)	0.89 (15.08)	0.16 (2.41)	0.94	3.14	60
331	Clay products	1.18 (1.76)	0.92 (9.36)	0.30 (1.48)	0.88	0.67	7.92
332	Glass	0.66 (1.12)	0.99 (11.87)	0.51 (1.69)	0.92	0.000 17	0.08
333	Ceramics	1.16 (2.40)	0.90 (12.59)	0.31 (2.05)	0.93	1.93	35.36
334	Cement	1.84 (1.69)	0.92 (6.15)	0.16 (0.92)	0.77	0.28	8.44
341	Iron and steel	2.14 (4.96)	0.81 (13.36)	0.12 (2.32)	0.93	9.70	960
342	Non-ferrous metals	0.78 (0.89)	1.01 (8.38)	0.46 (1.14)	0.88	0.006	0.10
350	Metal products	1.36 (2.20)	0.90 (10.26)	0.25 (1.61)	0.89	1.23	25
370	Electric machinery	1.69 (2.02)	0.87 (7.35)	0.18 (1.19)	0.80	1.22	67

Notes: *t*-values in parentheses. Implied $\hat{a} = 1/\exp(\hat{c}_1)$. Critical values: $\chi_1^2(0.05) = 3.84$; $\chi_1^2(0.01) = 6.63$; $\chi_2^2(0.05) = 5.99$; $\chi_2^2(0.01) = 9.21$.

hypothesis and warrants further investigation, but the important point to note is that it does not require the existence of a well-behaved aggregate production function, even though the production function may exist at firm level (as in Fisher's 1971 simulation), and that it explains the finding of $\hat{b}_1 < 1$.

IV. ACMS'S TIME SERIES REGRESSIONS

The analysis in the above section has been confined to production functions estimated using cross-sectional data. However, the argument follows through when time-series data are used and an allowance has to be made for technical progress. This case may also be illustrated with refer-

Table 2. *Summary statistics*

ISIC No.	Industry	a_{ave}	a_{max}	Country a_{max}	a_{min}	Country a_{min}	s	s/a_{ave}
202	Dairy products	0.39	0.51	Canada	0.24	Ceylon	0.09	0.23
203	Fruit and vegetable canning	0.42	0.59	Norway	0.26	Mexico	0.10	0.24
205	Grain and mill products	0.39	0.74	Norway	0.13	Colombia	0.13	0.35
206	Bakery products	0.49	0.62	Argentina	0.29	El Salvador	0.08	0.17
207	Sugar	0.39	0.57	Colombia	0.15	Japan	0.13	0.33
220	Tobacco	0.29	0.49	Denmark	0.08	Mexico	0.10	0.37
231	Textile-spinning and weaving	0.49	0.70	India	0.27	Ceylon	0.12	0.26
232	Knitting mills	0.53	0.75	Puerto Rico	0.33	Brazil	0.10	0.20
250	Lumber and wood	0.51	0.70	Ireland	0.30	Brazil	0.12	0.24
260	Furniture	0.59	0.70	Norway	0.42	Brazil	0.09	0.15
271	Pulp and paper	0.35	0.54	Colombia	0.16	Puerto Rico	0.10	0.28
280	Printing and publishing	0.53	0.78	Norway	0.39	Brazil	0.10	0.17
291	Leather finishing	0.51	0.66	Australia	0.29	Brazil	0.11	0.22
311	Basic chemicals	0.36	0.58	Ireland	0.20	Brazil	0.10	0.26
312	Fats and oils	0.32	0.44	United Kingdom	0.14	Ceylon	0.10	0.30
319	Miscellaneous chemicals	0.33	0.45	Ireland	0.24	Ceylon	0.06	0.19
331	Clay products	0.55	0.69	Japan	0.22	Puerto Rico	0.14	0.26
332	Glass	0.53	0.79	India	0.32	Colombia	0.10	0.23
333	Ceramics	0.62	0.86	Puerto Rico	0.40	Mexico	0.12	0.19
334	Cement	0.30	0.53	Australia	0.17	Mexico	0.11	0.37
341	Iron and steel	0.46	0.55	United States	0.27	Iraq	0.10	0.21
342	Non-ferrous metals	0.44	0.69	Colombia	0.30	Argentina	0.13	0.30
350	Metal products	0.52	0.66	El Salvador	0.25	Argentina	0.11	0.22
370	Electric machinery	0.48	0.65	Australia	0.21	Puerto Rico	0.13	0.27

ence to first the Cobb–Douglas and then the CES production function.

Differentiating the identity, $Q_t = w_t L_t + r_t K_t$, with respect to time and expressing the result in proportionate growth rates gives:

$$q_t = a_t \hat{w}_t + (1 - a_t)\hat{r}_t + a_t \ell_t + (1 - a_t)k_t \qquad (10)$$

where q, ℓ, k, $\hat{w}$, and $\hat{r}$ denote the growth rates of the respective variables. If $a_t\hat{w}_t + (1 - a_t)\hat{r}_t$ can be approximated by a constant trend rate of growth, λ, and if factor shares are constant, Equation 10 may be integrated to give the Cobb–Douglas relation:

$$Q_t = A_0\, \mathrm{e}^{\lambda t} L_t^a K_t^{(1-a)} \qquad (11)$$

Alternatively, as already seen, the CES production function was developed in order to accommodate, among other things, the possibility of nonconstant factor shares. If these show some pronounced trend over the period being considered, a more flexible functional form may prove a better approximation. One such function is the CES. This production function, with the assumption of constant returns to scale) is given by:

$$Q_t = \gamma_0\, \mathrm{e}^{\lambda t}[\delta L_t^{-\rho} + (1 - \delta)K_t^{-\rho}]^{-1/\rho} \qquad (12)$$

where, it will be recalled, δ is a distribution parameter and ρ is a parameter where the elasticity of substitution $\sigma = 1/(1 + \rho)$. λ is the rate of technical change. The CES function may be written using a Taylor series expansion as:

$$\ln Q_t = \ln \gamma_0 + \lambda_1 t + b_2 \ln L_t + b_3 \ln K_t + b_4(\ln K_t/L_t)^2 \qquad (13)$$

where $b_2 = \delta$, $b_3 = (1 - \delta)$; and $b_4 = -(1/2)\delta(1 - \delta)\rho$.

From Equation 13 it can be seen that if $\rho = 0$, $\hat{b}_4$ will equal zero and the equation will reduce to the Cobb–Douglas production function. It is one of the stylized facts of economic growth that over time there is a steady growth in the capital–labour ratio. Consequently, if the factor shares also exhibit a trend over the sample period, then the CES may prove to give a better fit, because its more flexible functional form allows this trend to be captured by the last two terms in Equation 13.[13] This fact, however, may have nothing to do with the value of an 'aggregate elasticity of substitution', which may not in fact exist. Fisher *et al.* (1977) used simulation analysis to answer the question of why and when aggregate CES production functions work well. The methodology followed that of Fisher (1971) for the Cobb–Douglas, in that they created individual firms which had well-defined CES micro production functions, but, because of aggregation problems, there was no aggregate CES production function.

[13] Estimates of the elasticity of substitution would be expected to be highly unstable with respect to the sample period chosen and this indeed proves to be the case. See Nerlove (1967) for an early summary.

Table 3. *Auxiliary regression* $\ln a_i = \mu_1 + \mu_2 \ln w_i$

ISIC No.	Industry	μ_1	μ_2	$\bar{R}^2$
202	Dairy products	−2.90 (−7.59)	0.28 (5.09)	0.62
203	Fruit and vegetable canning	−1.86 (−3.75)	0.14 (1.97)	0.18
205	Grain and mill products	−1.61 (−2.47)	0.09 (0.94)	−0.007
206	Bakery products	−1.30 (−3.26)	0.08 (1.47)	0.07
207	Sugar	−2.50 (−3.16)	0.22 (1.90)	0.18
220	Tobacco	−2.99 (−2.89)	0.24 (1.62)	0.10
231	Textile-spinning and weaving	−2.02 (−4.55)	0.19 (2.90)	0.30
232	Knitting mills	−2.10 (−4.84)	0.21 (3.36)	0.42
250	Lumber and wood	−1.61 (−3.72)	0.14 (2.14)	0.17
260	Furniture	−1.23 (−3.44)	0.10 (1.97)	0.16
271	Pulp and paper	−1.34 (−1.89)	0.035 (0.35)	−0.062
280	Printing and publishing	−1.56 (−3.89)	0.13 (2.31)	0.22
291	Leather finishing	−1.66 (−3.92)	0.14 (2.31)	0.21
311	Basic chemicals	−2.23 (−4.37)	0.17 (2.35)	0.23
312	Fats and oils	−2.30 (−3.71)	0.16 (1.79)	0.14
319	Miscellaneous chemicals	−1.84 (−4.43)	0.10 (1.77)	0.12
331	Clay products	−1.18 (−1.76)	0.08 (0.82)	−0.02
332	Glass	−0.66 (−1.12)	0.01 (0.01)	−0.09
333	Ceramics	−1.16 (−2.40)	0.10 (1.39)	0.08
334	Cement	−1.84 (−1.69)	0.08 (0.53)	−0.07
341	Iron and steel	−2.14 (−4.96)	0.19 (3.11)	0.42
342	Non-ferrous metals	−0.78 (−0.89)	−0.01 (−0.08)	−0.12
350	Metal products	−1.36 (−2.20)	0.09 (1.11)	0.019
370	Electric machinery	−1.69 (−2.02)	0.13 (1.10)	0.016

Notwithstanding this, the data gave a good fit in terms of the conventional diagnostics when the aggregate production function was fitted. The authors concluded, however, that they could not determine what they termed the 'organizing principle' which would make the CES aggregate production function give good predictive power when the underlying microproduction functions were CES but aggregation was not theoretically possible. It can be seen, first, why the aggregate relationship had to produce good results by reconsidering Fisher's (1971) simulation experiments, as he provided some interesting insights as to why these paradoxical results appear (the arguments with the Cobb–Douglas are more intuitive). Next, the organizing principle behind the CES will be demonstrated.

Fisher assumed a small number of firms whose individual production functions were Cobb–Douglas, but with different output elasticities. The conditions for successful aggregation were explicitly violated. The simulation was run over 20 time periods during which the aggregate labour force, the level of technology, and the firms' capital stocks were assumed to grow at a constant rate, subject to small random fluctuations. Labour was allocated between firms to optimize output. Thus, labour was paid its marginal product and the wage rate was determined by the supply and demand schedules for labour. Fisher then ran regressions on the resulting aggregate data, estimating the Cobb–Douglas production function and seeing how well such a function predicted wages. He generally found very high R^2s and a high degree of accuracy in the predictive ability of the Cobb–Douglas, to his evident surprise. Why does the aggregate Cobb–Douglas production function 'work' in giving such good statistical fits?

Consider the simplest case of an economy of two firms, where the underlying production function of each is a Cobb–Douglas, markets are competitive, and factors are paid their marginal products. The production functions have different values for the output elasticities, and these are equal to their respective factor shares. It is known that the production functions cannot be summed to give an aggregate Cobb–Douglas production function. Nevertheless, because of the underlying identity, it can be shown that, in certain circumstances, the aggregate Cobb–Douglas will, in fact, give a good fit to the data, as Fisher (1971) discovered. The output, measured in terms of value added, of the two firms may be written in terms of the identity as:

$$Q_{1t} \equiv w_{1t}L_{1t} + r_{1t}K_{1t} \tag{14a}$$

$$Q_{2t} \equiv w_{2t}L_{2t} + r_{2t}K_{2t} \tag{14b}$$

Sum Equations 14*a* and 14*b* arithmetically. If the aggregate shares are constant, then, as seen from the discussion concerning Equations 5 and 6, the data will give an exact fit to the aggregate Cobb–Douglas production function. Aggregate labour's share is given by $\Sigma a_i Q_i/Q (i = 1, 2)$ where Q is the combined value added of the two firms. If the output shares do not change greatly over time,[14] or, for a large number of firms, any changes are not correlated with the labour share, so that there is no systematic relationship between the growth rate of the firms and the size of their corresponding labour shares, then the aggregate labour share will be approximately constant. If this is the case, the Cobb–Douglas relationship will give a good fit to the aggregate data. In other words, if the Cobb–Douglas relationship 'works', it must be because factor shares are constant. But as Fisher (1971, p. 306) concluded: 'the view that the constancy of the labour share is due to the presence of an aggregate Cobb–Douglas production function is mistaken. Causation runs the other way around and the apparent success of aggregate Cobb–Douglas production functions is due to the relative constancy of labour's share.' The reason why this is the case is simply the workings of the underlying accounting identity (for a more detailed discussion of Fisher, 1971, see Shaikh, 1980).[15, 16]

The same general arguments apply to the CES. If the data are summed, the high degree of correspondence between Q, L, and K from the identity will inevitably give a high R^2, when aggregate data are used in the regression analysis. If the individual shares are trended, then the aggregate share is also likely to be a function of time. If this is the case then one should expect a more flexible form such as Equation 12 to give a better statistical fit to the aggregate data.

This point may be made in a slightly different way. To this purpose, return to the CES Equation 12 and express it in growth rates:

$$q_t = \lambda_t + \left[\frac{\delta L_t^{-\rho}}{V_t}\right]\ell_t + \left[\frac{(1-\delta)K_t^{-\rho}}{V_t}\right]k_t \tag{15}$$

where:

$$V_t = [\delta L_t^{-\rho} + (1-\delta)K_t^{-\delta}] \tag{16}$$

Now consider again the accounting identity in time series form given by Equation 10, namely:

$$q_t = a_t\hat{w}_t + (1-a)\hat{r}_t + a_t\ell_t + (1-a_t)k_t \tag{17}$$

Assuming $a_t\hat{w}_t + (1-a_t)\hat{r}_t = \varphi$ (which is a constant), Equation 17 becomes:

$$q_t = \varphi + a_t\ell_t + (1-a_t)k_t \tag{18}$$

Since the identity holds by definition, comparing Equation 15 with Equation 18 it is concluded that the following relations must hold:

$$\lambda_t = \varphi \text{ (a constant)} \tag{19}$$

$$a_t = \delta L_t^{-\rho}/V_t \tag{20}$$

$$(1-a_t) = (1-\delta)K_t^{-\rho}/V_t \tag{21}$$

Equations 20 and 21 can also be derived from the CES production function and the marginal productivity theory of factor pricing. But the fact that these relationships are found to hold empirically cannot be used as confirmation of the marginal productivity conditions, because the argument shows that a CES aggregate production function may not exist, and yet if the data happen to follow the paths given by Equations 19–21, the CES will give a good statis-

[14] This was true in Fisher's simulations.

[15] The simulation analysis of Felipe and Holz (2000) has shown that the Cobb–Douglas production function will still perform well in spite of quite large random fluctuations in the factor shares.

[16] It is important to note that it is not necessary for each firm to have a well-defined Cobb–Douglas production function. All that is for the firms' factor shares to be constant, because of, say, a constant markup pricing policy.

tical fit. To see this, substitute Equations 19–21 into 18 and integrate. This will yield Equation 12.

From this analysis it can be inferred that the 'organizing principle', in terms of the behaviour of the factor share, that explains why and when the CES production function Equation 12 will produce good results is given by the paths in Equations 20 and 21 (together with Equation 19). It must be stressed that the reason for following such paths need not necessarily be the neoclassical theory of factor pricing and the assumptions of profit maximization and competitive markets. If the labour share is changing over time, a more flexible form such as Equation 12 should provide a good approximation to the data. Furthermore, if the shares and φ follow exactly the paths given by Equations 19, 20, and 21, then expression 12 will be exactly the accounting identity. Nevertheless, one could ask whether there is any other theoretical justification for the factor share paths Equations 20 and 21. Assume, for example, that both capital and labour grow at constant rates, with the former growing faster than the latter. In this economy also assume that the technology cannot be represented by a CES production function. Also assume that due to sociopolitical factors, e.g., trade union militancy, labour's relative share (i.e., $a_t/(1-a_t)$) is also increasing at a constant rate over time. As both labour's relative share and the capital–labour ratio are increasing over time, they may well be related to each other by:

$$\left(\frac{\mathrm{d}a/\mathrm{d}t}{a}\right)_t - \left(\frac{\mathrm{d}(1-a)/\mathrm{d}t}{(1-a)}\right)_t = b(k_t - \ell_t) \tag{22}$$

where b is a constant.

However, this expression is identical to the one obtained by subtracting the growth rates of the factor shares provided by the CES production function together with the marginal productivity theory of factor pricing (Equation 20, expressed in growth rates, minus Equation 21, also in growth rates). But the example provided above has nothing to do with the CES production function. The rate of change of labour's relative share is not caused by the growth of the capital–labour ratio and the CES does not describe the underlying technology of the economy.[17]

ACMS also estimated the CES production function using the time series data from Solow's (1957) classic study. This was for private non-farm GDP for the US over the period 1909–49. Clearly, technology changed over this period, and ACMS assumed it was at a constant rate and proxied it by a time trend (Equation 34 in ACMS). Under these circumstances, with a little manipulation, Equation 8 may be written as (substituting the subscript i for t):

$$\begin{aligned}\ln a_t &= \ln(w_t L_t/Q_t)\\ &= [\sigma \ln(\delta) + (\sigma-1)\ln\gamma_0] + (1-\sigma)\ln w_t - \lambda(1-\sigma)t\end{aligned} \tag{23}$$

where it is assumed that technical change grows at a constant exponential rate so $\gamma_t = \gamma_0 e^{\lambda t}$. The term in square brackets is a constant and, under neoclassical assumptions, λ measures the rate of growth of total factor productivity. Equation 23 is estimated as:

$$\ln a_t = c_3 + b_6 \ln w_t + b_7 t \tag{24}$$

Using Solow's corrected data (Hogan, 1958), a rerun of ACMS's regression gives the following result:

$$\begin{aligned}\ln a_t &= -0.062 + 0.387\ln w_t - 0.0069t\\ &\quad\;(-1.29)\quad (7.48)\qquad (-7.22)\\ \bar{R}^2 &= 0.574,\quad \mathrm{DW} = 1.802,\quad \mathrm{SEE} = 0.018\end{aligned}$$

Since this is merely estimating an identity the usual tests for the orders of integration, etc. were not undertaken, but the Durbin–Watson suggests that the variables are cointegrated. Lags or specifying an error correction model were also not considered. While ACMS's results were not precisely replicated, the estimate of the 'aggregate elasticity of substitution' of 0.387 is sufficiently close to their estimate of 0.431. However, these results do not suggest that *if* there is an underlying aggregate production function, it cannot be a Cobb–Douglas, *pace* ACMS. (ACMS rejected the hypothesis of a Cobb–Douglas production function because the coefficients of ln$w$ and t are significantly different from zero). To see why this is the case, first consider the identity given by the labour share:

$$\ln a_t \equiv 1.0\ln w_t - 1.0\ln(Q_t/L_t) \tag{25}$$

One way of interpreting Equation 24 is that it is merely Equation 25 with $\ln(Q_t/L_t)$ proxied by the time trend, t. $\ln(Q_t/L)_t$ is strongly trended, as may be seen from the following regression:

$$\begin{aligned}\ln(Q_t/L_t) &= -0.504 + 0.018t\\ &\quad\;(-41.73)\;(36.17)\\ \bar{R}^2 &= 0.970,\quad \mathrm{DW} = 0.752,\quad \mathrm{SEE} = 0.0379\end{aligned}$$

However, w_t grows at a trend rate almost identical to that of Q_t/L_t:

[17] An additional argument is that the CES can derived from a Box–Cox transformation, as Zarembka (1974) showed. (See also Bairam, 1997.) This is simply a mathematical transformation of the data with a view to improving the statistical goodness of fit by letting the data select the functional form, but which does not necessarily reflect any technological relationships.

$$\ln w_t = -0.922 + 0.0183t$$
$$(-52.81)\ (25.05)$$
$$\bar{R}^2 = 0.686, \quad DW = 0.686, \quad SEE = 0.0548$$

Thus, there is strong multicollinearity between $\ln w_t$, $\ln(Q_t/L_t)$, and t, which is sufficient to bias the coefficient of $\ln w$ below unity and inflate the standard errors of the estimated coefficients. If one was in fact merely estimating the identity given by Equation 25, as the sum of its coefficients are equal to zero, it would be expected that if we were to proxy $\ln w_t$ by its trend rate of growth of $0.0183t$ in the equation $\ln \alpha_t = -0.062 + 0.387 \ln w_t - 0.0069t$, then the sum of the coefficients of t should also equal zero. As $0.0183 \times 0.387 = 0.0071 \approx 0.0069$, this indeed proves to be the case. In other words, $\ln a_t$ does not show any significant change over time. If we were to interpret these results as reflecting a production function, we would have to conclude it was a Cobb–Douglas and not a CES.

Alternatively, this may be confirmed by regressing $\ln a_t$ directly on a time trend, where the slope coefficient should not be significantly different from zero. This, not surprisingly in view of the above results, proves to be true:

$$\ln a_t = -0.419 + 0.309 \times 10^{-4} t$$
$$(-47.84)\ \ (0.09)$$
$$\bar{R}^2 = -0.025, \quad DW = 1.342, \quad SEE = 0.0275$$

Consequently, under the neoclassical assumptions, the Cobb–Douglas production function cannot be ruled out. However, it is worth re-emphasizing that all we are doing is confirming an identity.

ACMS proceeded to estimate another side relation which does not suffer from the problem of multicollinearity, and somewhat ironically found that the results did not refute the hypothesis of a unity 'elasticity of substitution'. The equation was (Equation 38 in ACMS):

$$\ln(a_t/(1 - a_t)) = \ln[\delta/(1 - \delta)] + \rho \ln(K/L)_t \qquad (26)$$

Re-estimating the equation with Solow's data gives:

$$\ln(a_t/(1 - a_t)) = 0.750 - 0.096 \ln(K/L)_t$$
$$(7.88)\ (-0.97)$$
$$\bar{R}^2 = -0.001, \quad DW = 1.356, \quad SEE = 0.0778$$

which is close to the value of the slope coefficient found by ACMS (which was −0.095). The estimate of σ is given by $1/(1 + \hat{\rho})$ takes a value of 1.106, which is not significantly different from unity at the 5% confidence level (the t-value for this null hypothesis is 0.88). As ACMS point out, if the relationships were exact including that of the ACMS 'production function', the estimated values of δ and ρ from Equation 26 should be exactly the same as those obtained from estimating Equation 23. 'Hence the discrepancy must be due to the different assumptions about the errors implicit in the statistical estimation methods. This problem remains an open one at the moment' (ACMS, p. 246). The resolution of this problem is, as shown here, the fact that there is significant multicollinearity in the estimation of Equation 24.

The parsimonious explanation of these regressions is that, because of the accounting identity, all they show is that factor shares vary little over time, due for example, to a constant aggregate mark-up. The data are compatible with a Cobb–Douglas production function, with all the usual neoclassical assumptions, but the data provide no independent evidence that such a production function actually exists. The data cannot be used to test these assumptions or whether or not there is a well-behaved aggregate production function.

V. CONCLUSIONS

This paper has shown that the empirical evidence provided by Arrow *et al.* (1961) in their seminal work on the CES production function, may be interpreted merely as, in Simon's words, a statistical artefact. The reason is that Occam's razor suggests that the CES production function, as well as the side equations ACMS derived, merely reflect the underlying accounting identity that value added equals the wage bill plus profits. All their empirical results and arguments are more parsimoniously explained with reference to this identity than with reference to an underlying aggregate technology. Since the explanation provided in terms of the accounting identity encompasses that in terms of an aggregate production function, this poses serious problems for the legitimacy and conclusions of ACMS's work, as well as for the subsequent work during the last 40 years using this aggregate form.

REFERENCES

Arrow, K., Chenery, H. B., Minhas, S. and Solow, R. M. (1961) Capital-labour substitution and economic efficiency, *Review of Economics and Statistics*, **XLIII**, 225–50.

Bairam, E. I. (1997) *Homogeneous and Nonhomogeneous Production Functions. Theory and Applications*, Avebury, Aldershot.

Felipe, J. (2000) On the myth and mystery of Singapore's 'zero TFP', *Asian Economic Journal*, **14**, 187–209.

Felipe, J. and Holz, C. (2000) Why do aggregate production functions work? Fisher's simulations, Shaikh's identity and some new results', *mimeo*, Georgia Institute of Technology.

Fisher, F. M. (1969) The existence of aggregate production functions, *Econometrica*, **37**, 553–77.

Fisher, F. M. (1971) Aggregate production functions and the explanation of wages: a simulation experiment, *Review of Economics and Statistics*, **LIII**, 305–25.

Fisher, F. M. (1987) Aggregation problems, in *The New Palgrave. A Dictionary of Economics*, (Eds) J. Eatwell, M. Milgate and P. Newman, Macmillan, Basingstoke, pp. 53-7.

Fisher, F. M., Solow R. M. and Kearl J. M. (1977) Aggregate production functions: some CES experiments, *Review of Economic Studies*, **XLIV**, 305–20.

Fuchs, V. R. (1963) Capital and labor substitution: a note, *Review of Economics and Statistics*, **XLV**, 436–8.

Hogan, W. P. (1958) Technical progress and production functions, *Review of Economics and Statistics*, **40**, 407–11.

Marshak, J. and Andrews, W. H. (1944) Random simultaneous equations and the theory of production, *Econometrica*, **12**, 143–205.

McCombie, J. S. L. (1998) Are there laws of production?: an assessment of the early criticisms of the Cobb–Douglas production function, *Review of Political Economy*, **10**, 141–73.

McCombie, J. S. L. (2000) Solow's residual, technical change and aggregate production functions, *Journal of Post Keynesian Economics*, (forthcoming).

McCombie, J. S. L. and Dixon, R. J. (1991) Estimating technical change in aggregate production functions: a critique, *International Review of Applied Economics*, **5**, 24–46.

Nerlove, M. (1967) Recent empirical studies of the CES and related production functions, in *The Theoretical and Empirical Analysis of Production*, (Ed) Murray Brown, National Bureau of Economic Research, New York, 55–122.

Phelps Brown, E. H. (1957) The meaning of the fitted Cobb–Douglas function. *Quarterly Journal of Economics*, **71**, 546–60.

Shaikh, A. (1974) Laws of production and laws of algebra: the humbug production function, *Review of Economics and Statistics*, **LVI**, 115–20.

Shaikh, A. (1980) Laws of production and laws of algebra: humbug II, in *Growth, Profits, and Property, Essays in the Revival of Political Economy*, (Ed) E. J. Nell, Cambridge University Press, Cambridge, pp. 80–95.

Simon, H. A. (1979a) On parsimonious explanations of production relations, *Scandinavian Journal of Economics*, **81**, 459–74.

Simon, H. A. (1979b) Rational decision making in business organizations, *American Economic Review*, **69**, 493–513.

Simon, H. A. and Levy, F. K. (1963) A note on the Cobb–Douglas function, *Review of Economic Studies*, **30**, 93–4.

Solow, R. M. (1957) Technical change and the aggregate production function, *Review of Economics and Statistics*, **39**, 312–20.

Solow, R. M. (1974) Laws of production and laws of algebra: the humbug production function: a comment, *The Review of Economics and Statistics*, **LVI**, 121.

Walters, A. A. (1963) Production and cost functions, *Econometrica*, **31**, 1–66.

Zarembka, P. (1974) Transformation of variables in econometrics, in *Frontiers in Econometrics*, (Ed) P. Zarembka, Academic Press, New York and London, 81–104.