Cambridge Journal of Economics 2003, 27, 695–721

Some methodological problems with the neoclassical analysis of the East Asian miracle

Jesus Felipe and J. S. L. McCombie*

This paper discusses the recent controversy over the sources of economic growth in East Asia. This empirical work has either used growth accounting or estimated econometrically aggregate production functions. It is shown that it is possible to approximate the value-added accounting identity (i.e., value added equals labour's compensation plus total profits) by a mathematical expression that has all the properties of a well-behaved neoclassical aggregate production function. This implies that statistical estimations of putative aggregate production functions can provide no independent evidence as to whether they accurately describe the production technology of the economy or, indeed, whether the aggregate production function actually exists. A corollary is that the conventional measures of the growth of total factor productivity cannot be unambiguously interpreted as an estimate of the rate of technical progress. The paper reviews the works of Kim and Lau and Young and, in the light of this, explains why both analyses and interpretations of the notion of total factor productivity growth as the rate of technical progress are problematical.

Key words: Aggregate production function, East Asian miracle, Equifinality Theorem, Income accounting identity, Total factor productivity growth
JEL classifications: O40, O50, E23, B41

1. Introduction

This paper considers the controversy that has arisen over the contribution of total factor productivity (TFP) growth to the recent fast output growth of the East Asian economies. It is argued that the conventional neoclassical methodology is fundamentally flawed and that this has serious consequences for the orthodox interpretation of TFP growth.

Manuscript received 18 September 2000; final version received 19 November 2001.
*Asian Development Bank, Manila, Philippines, and Downing College, Cambridge, respectively. We are grateful to Mike Intriligator, Richard Nelson, the participants of the Project LINK meeting conference in Kuala Lumpur, the participants in the economics seminars of the Asian Development Bank, the National University of Singapore, the Hong Kong University of Science and Technology, the University of Hong Kong, the City University of Hong Kong, Universidad Carlos III de Madrid, Georgia Institute of Technology, and the participants in the session on 'Productivity, Economic Growth and Convergence: The Experience of Asia' of the 3rd Biennial Conference of the Western Economic Association (Bangkok, January 1998) for their valuable comments on successive versions of the paper. A referee also provided us with helpful suggestions. Jesus Felipe acknowledges financial support from the Center for International Business Education and Research (CIBER) at the Georgia Institute of Technology. JSL McCombie is grateful to the members of the Department of Economics, University of Otago, where he was William Evans Visiting Fellow from July to September 2001. This paper represents the views of the authors and should not be interpreted as reflecting those of the Asian Development Bank, its executive directors or the countries that they represent. The usual disclaimer applies.

The conventional wisdom (see, for example, Page, 1994) is that much of the success of the East Asian economies (Hong Kong, Singapore, South Korea and Taiwan) is largely due to technological catch-up. However, this has recently been challenged by Young's (1992, 1994A, 1994B, 1995) careful and detailed growth accounting studies of these countries. The relatively high per capita growth rates that these countries experienced become progressively less remarkable once an allowance is made for the growth of the labour input adjusted for changes in participation rates, etc., and the fast rate of capital accumulation (see Krugman, 1994, for a discussion of the implications of these results). In fact, the growth rates of TFP of the East Asian Tigers turn out to be no higher than those of many other developing countries; and the calculations suggest, for example, that Singapore has experienced no growth in TFP from the mid-1960s. This finding has been confirmed by the explicit estimation of the production function for these countries (Kim and Lau, 1994). Nevertheless, many find that these conclusions are difficult to accept (Felipe, 1999).[1]

The debate about the sources of growth in East Asia has been almost exclusively empirical in nature, in that it has been driven by these surprising findings of Young and Kim and Lau. Indeed, it is likely that had these authors found significant TFP growth rates for the East Asian countries, their findings would not have proved to be so controversial. Broadly speaking, there have been two reactions to their results.

The first was simply to accept the results and policy implications discussed by Young, Krugman and Kim and Lau as accurate. Few scholars (certainly very few in East Asia) have embraced this view (see Rodrigo, 2000).

The second reaction was to question the results. There have been two lines of approach. One is based on the argument that the results of Young and Kim and Lau are empirically subject to large errors. A number of authors have simply re-done the exercises (in particular growth accounting) for these countries with different data sets, using different values for the factor shares and for slightly different time periods (see Felipe, 1999, for a survey of this literature and the various estimates of TFP growth). This has led to some modification of the results, but this has not been universally seen as refuting the main conclusions of Young's and Kim and Lau's studies, which is perhaps not surprising given the careful nature of their work. Consequently, the implausibility of the low TFP growth rates continues to be the focal point in discussions about the sources of growth in the East Asian economies.

The alternative approach has been to question the methodology that Young and Kim and Lau used in obtaining their results, although remaining within the neoclassical framework. In other words, how sound are the particular assumptions underlying the growth accounting approach and the production function estimation that was used in their respective studies? This is the approach of Nelson and Pack (1999). They showed that if technical change is biased, the rate of technical progress that occurs throughout the period under consideration will affect the values of the output elasticities of labour and capital if the elasticity of substitution differs from unity (see also Nelson, 1973). Thus, they argue that the value of the capital share used in the growth accounting studies of these countries

[1] Pack and Page (1994A, p. 253), commenting on Young's work, argue: 'Totally autarchic and corrupt Burma exceeds the TFP growth of South Korea! . . . Bangladesh, the site of the Indo-Pakistani war of the early seventies, the separation of West from East Bengal, and devastating cyclones, has greater TFP growth than Japan, as does Uganda, dominated for much of the period by Idi Amin, incessant civil war and the killing of much of the educated population . . . These results strain credulity and severely undermine the claim, based on Young's capital stock, that the HPAE [high performing Asian economies] were simply run-of-the-mill countries in the period from 1970 to 1985.'

(taken from the national accounts) is theoretically incorrect and empirically takes too high a value. The use of a lower capital share generates a higher TFP rate in a growth accounting exercise, given that the capital–labour ratio is growing.[1] Thus, Young's results will understate the true contribution of TFP growth to that of output.

While Nelson and Pack have raised some important questions over the results obtained, their critique of the growth accounting approach has been squarely within the neoclassical orthodoxy. They accept the standard growth accounting assumptions, i.e., profit maximisation and competitive markets, as well as the existence of a well-behaved aggregate production function (Nelson and Pack, 1999, p. 424).[2] The last assumption is of crucial importance, for it is the *sine qua non* of the growth accounting exercises, and yet it has never been questioned in the debate, although it is easy to understand why this is the case. While the hypothesis of competitive markets can be relaxed without too much difficulty (it leads to a slightly different growth accounting equation), if the aggregate production function does not exist, the whole exercise, either growth accounting or econometrically estimating aggregate production functions, becomes a pointless endeavour.

The Cambridge Capital Theory Controversies have raised important theoretical questions about the aggregate production function (Harcourt, 1972), but these have largely been dismissed on the grounds that the aggregate production function usually gives plausible empirical results.[3] There are also aggregation problems. Does it makes sense in a less developed country to sum the production technologies of the peasant farmer, the small over-crowded sweat shop and the modern factory of the multinational corporation into a single aggregate production relationship? This is, of course, implicit in the construction of the aggregate production function for the whole economy, the industrial sector or even for industries at the two-digit SIC or higher level. What is important is that the theoretical results for the necessary conditions for the successful aggregation of micro-production functions are so stringent that one cannot expect any actual economy to satisfy them. Consequently, an aggregate production function cannot be used even as an approximation to reality (Fisher, 1969, 1993; Felipe and Fisher, 2003). As Fisher put it: 'There can be no presumption that our exact conditions don't matter simply because one is interested in reasonable approximations over a limited domain. Indeed, I think the presumption must be the other way' (Fisher, 1969, p. 571).[4]

This problem is also likewise summarily dismissed. Any theory makes unrealistic assumptions; indeed, that is what makes it a theory. What matters, according to this instrumentalist argument, is the predictive value of the model (Friedman, 1953).[5]

[1] See Felipe and McCombie (2001) for an evaluation and empirical implementation of Nelson and Pack's (1999) arguments.

[2] However, they also develop an alternative two-sector model where each sector exhibits fixed coefficients technology. Development occurs through the shifting of resources from the 'craft' to the 'modern' sector. The rate at which this occurs is a function of the level of entrepreneurship in the country.

[3] What is 'plausible' to a certain degree begs the question, but it is normally taken to be that the regressions give a good statistical fit in terms of the usual diagnostics with the estimated output elasticities not greatly different from the relevant factor shares. As Fisher (1971) has noted, Solow once remarked that if Douglas had found capital's output elasticity to be three-quarters rather than one-quarter, we should not now be talking about production functions. Recent developments in endogenous growth theory, however, could lead to a modification of this view.

[4] The most general conditions state that the existence of an aggregate capital stock requires that the production functions of individual firms differ at most by capital-augmenting technical change. It is also necessary for the existence of a labour aggregate that every firm hires the same proportions of each type of labour. Finally, the existence of an aggregate of output requires that every firm produce the same market basket of outputs.

[5] This is a controversial methodological position that we shall not pursue here.

However, this paper argues that the good empirical fits that are often found for output–input data are no guarantee that the aggregate production function actually reflects the underlying technology of the economy or, indeed, that the aggregate production function even exists. In these circumstances, the validity of the growth accounting approach, which relies on the assumption of a well-behaved production function, must be seriously questioned.

As we shall show, the problem arises from the combination of two related issues. First, underlying all putative aggregate production functions is an income accounting identity that relates the value of output to the sum of the wage bill plus total profits. Through an algebraic transformation, this can be rewritten in a form that looks like a production function. Second, the measure of output used in these aggregate exercises is not a physical quantity, but a measure of (constant price) value. The only possible way to express aggregate output (e.g., the sum of corn and iron) is in value terms. Value added, however deflated or expressed in terms of an index, is a value measure and not a physical quantity, even though it is often referred to as the 'volume' of output. The use of value magnitudes precludes the possibility of obtaining any indication of the underlying technology of the 'representative firm' via the statistical estimation of a production function.

The plan of the paper is as follows. In Section 2, we consider the question of whether or not, because of an underlying accounting identity, the production function can be considered to be a behavioural relationship in the sense of being capable of empirical refutation. In undertaking this, we discuss and extend the analyses of Phelps Brown (1957), Shaikh (1974, 1980, 1987) and Simon (1979A). It is surprising the extent to which this important critique of the aggregate production function has been ignored in the literature.[1] Next, we provide a generalisation of this result and discuss its important implications. In Section 3, we further develop the argument in the context of the work of Kim and Lau (1994) on the Asian Newly Industrialised Economies (NIEs). We argue that the conventional interpretation of their estimates of the production function is questionable. Section 4 provides an empirical illustration of the critique. In Section 5, we re-examine the growth accounting approach of Young (1992, 1995), and his cross-country econometric work (Young, 1994B). Section 6 concludes.

2. The aggregate production function and the accounting identity

The microeconomic production function is a technological relationship between quantities of output and inputs.[2] In its general form, it may be expressed as $Q = F(L, K', t)$, where Q is (physical) output, L is the labour input, K' is the capital stock expressed in physical units, such as the number of identical machines, and t is time, which captures exogenous technical change. Assuming neutral technical progress, the production function may be expressed as $Q = A(t)f(L, K')$, where the shift parameter $A(t)$ represents the level of technology. Alternatively, when technical change is factor augmenting, output is given by $Q = F(A_L L, A_{K'} K')$, where A_L and $A_{K'}$ are the levels of the efficiency of L and K' and grow at the rates λ_L and $\lambda_{K'}$.

[1] The neglect of these issues in the literature is surprising, since Simon (1979B) thought they were sufficiently important that he raised them in his Nobel Laureate lecture. McCombie (1998) explores why this may be the case. An important exception is Solow (1974, 1987), but see Shaikh (1980) and McCombie (2001).

[2] However, May (1947, p. 63) argued that even the firm's production function is not a purely technical relationship, as it results from an optimising decision-making process. In other words, it is maximum level of output that can be obtained from any given level of inputs and technology.

Taking logarithms and differentiating $Q = F(L, K', t)$, a general specification of the production function, with respect to time gives:

$$q_t = \lambda_t + \alpha_t l_t + \beta_t k'_t \tag{1}$$

where q, l, k' and λ denote the exponential growth rates of output, labour, capital and the rate of technical progress respectively. The output elasticities of labour and capital, α_t and β_t, have time subscripts because they may change over time, depending upon the elasticity of substitution and the relative growth rates of output, labour and capital. If the assumptions of profit maximisation and competitive markets are invoked, then the relevant marginal productivities F_L and $F_{K'}$ are equal to the relevant factor prices measured in terms of output, and the elasticities equal the factor shares.[1] This forms the basis of the growth accounting approach, where the values of the output elasticities are taken to be equal to the observed factor shares. The simplest approach is to calculate the growth of TFP using equation (1). More sophisticated analyses attempt to whittle down the 'residual' (λ_t) by including other sources of growth such as increasing returns to scale, the increase in the skills of the labour force, the intersectoral transfer of labour from low (agriculture) to high (manufacturing) productivity sectors, the contribution of R&D and the effect of the diffusion of technology (see, for example, Denison, 1962, 1967, 1985). Referring to equation (1) again, it can be seen that the basis of the method is that it is considered meaningful to ascribe a certain proportion of output growth separately to the growth of the labour and the capital inputs and the remainder to technical change. If, on the other hand, one opts for econometric estimation, it is necessary to choose a specific functional form for equation (1), such as the Cobb–Douglas, constant elasticity of substitution (CES) or translogarithmic (translog) production function. These are often directly estimated in order to determine the rate of technical progress (neutral or factor augmenting), the elasticity of substitution, etc.

The problem with the above framework is that, in most empirical analyses, including those concerning the East Asian miracle, the series of output and inputs used are not physical quantities, but deflated monetary values, i.e., constant price value data, or what are somewhat misleadingly termed 'quantity' indices.[2] The reason is that, even at low levels of aggregation, such as the four-digit SIC, output and capital cannot be measured as physical quantities. While this is perhaps self-evident, it nevertheless has important ramifications.[3] This is because there is an underlying accounting identity that precludes any testing of the putative production function with value data, as we shall see below. It is useful to begin by considering the simplest case, namely the Cobb–Douglas relationship, and then we shall generalise the argument to any production function.

2.1 The Cobb–Douglas case

Following Shaikh (1974, 1980) and Simon (1979A), let us consider the accounting identity that relates value added to the total wage bill and total profits:

[1] When markets are not perfectly competitive, the values of the factor shares and the relevant output elasticities will diverge.

[2] There have been very few studies that estimate 'engineering' production functions using physical data. See Wibe (1984) for a survey.

[3] Despite this, Ferguson argued: 'Neoclassical theory deals with macroeconomic aggregates, usually by constructing the aggregate theory *by analogy* with the corresponding microeconomic concepts. Whether or not this is useful is an empirical question to which I believe an empirical answer can be given [...] It would be better to say that aggregate analogies provide working hypotheses for econometricians' (Ferguson, 1971, pp. 251–2; emphasis added).

$$VA_t \equiv w_t L_t + r_t K_t \qquad (2)$$

where VA, w, L, r and K denote output (real value added), the average real wage rate, employment, the average rate of return per cent per annum, and the stock of capital in value terms. A similar argument can be developed using gross output and including intermediate materials on the right-hand side of equation (2). The data in equation (2) may be derived from the national accounts, and the same identity holds at any level, including that of the firm. In practice, the series of output and inputs used to estimate aggregate production functions are those in equation (2), i.e., output is real value added, and the stock of capital is measured in constant price monetary values, usually calculated using the perpetual inventory method. As indicated above, this is because the only way to aggregate heterogeneous physical quantities is by expressing them in value terms, i.e., by using prices. This is the problem Robinson (1953–54) brought up concerning the units in which capital should be measured.[1] The only possible physical quantity in this framework is labour (either the number of homogeneous workers or their total hours worked).[2] But even here, the existence of an aggregate of heterogeneous labour in the production function requires certain conditions to be fulfilled (see Fisher, 1969). This makes the empirics of the production function rather problematic.

It is worth emphasising that, contrary to the arguments in Hulten (2000, pp. 11–12), there are no behavioural assumptions, such as perfect competition and constant returns, or indeed even the existence of a well-defined production function, underlying equation (2). This equation simply states that total income is divided between labour income (the product of the average wage rate and employment) and capital income (the product of the average profit rate times the value of the stock of capital). The former consists of wages and salaries, and the latter includes profits, rental payments and interest payments for all types of capital goods.[3] As Samuelson (1979, p. 932) cogently put it: 'No one can stop us from labelling this last vector [residually computed profit returns to "property" or to the nonlabor factor] as (rK_j), as J. B. Clark's model would permit—even though we have no warrant for believing that noncompetitive industries have a common profit rate r and use leets capital (K_j) in proportion to the $(p_jQ_j - w_jL_j)$ elements!'[4,5] It would not matter for purposes of the argument if the identity were written in terms of the user cost of capital as in Jorgenson and Griliches (1967) or Jorgenson *et al.* (1987) (as is standard in neoclassical economics). In this case, constant returns to scale and the conditions for producer equilibrium are assumed to hold. It will become clear below why this is not a problem: the use of the data will always indicate constant returns and that the conditions for producer equilibrium hold whatever the true state of affairs.

[1] In a later paper, Robinson (1971) asked: 'In what unit is capital to be measured? The figures in the time-series are collected in the first instance in terms of dollars; however they may be deflated or adjusted, the amount of capital in the statistics is a sum of *value*. How can this be made to correspond to a physical factor of production?' (Robinson, 1971, p. 598; emphasis added). After the Cambridge Capital Theory debates died out, the references to these issues in the literature are almost non-existent. Bernanke (1987) is an exception.

[2] However, as we have noted, labour is sometimes adjusted for quality, using relative wages as weights as a proxy for different skills.

[3] Hulten (2000, pp. 11–12) argues that Jorgenson and Griliches (1967) needed to assume and impose constant returns to estimate the return to capital residually from the identity (the procedure we adopt as well). This is not correct. As we have indicated, the value-added identity is independent of the notion of returns to scale. Hulten, as well as Jorgenson and Griliches, are linking the identity to the production function via Euler's Theorem and the neoclassical theory of factor pricing, and the latter does require this assumption. Nevertheless, the identity, *per se*, must always hold regardless of the underlying assumptions.

[4] The notation has been changed to make it consistent with that in this paper. The subscript j denotes the jth firm. p_j is the price of output Q_j.

[5] Leets is simply steel spelled backwards. The term is due to Joan Robinson, commenting on Meade's use of 'steel' for capital.

Totally differentiating (2) with respect to time, and expressing the identity in terms of exponential growth rates, we obtain:

$$va_t = a_t\varphi_{wt} + (1 - a_t)\varphi_{rt} + a_t l_t + (1 - a_t)k_t \quad (3)$$
$$= \varphi_t + a_t l_t + (1 - a_t)k_t$$

where va, l and k are the growth rates of real value added, labour and capital. The variables φ_{wt} and φ_{rt} are the growth rates of the wage and profit rates. Labour's and capital's shares in value added are given by $a_t = w_t L_t / VA_t$ and $(1 - a_t) = r_t K_t / VA_t$ and φ_t equals $a_t\varphi_{wt} + (1 - a_t)\varphi_{rt}$. Without affecting the substance of the argument, we assume initially that factor shares are constant, i.e., $a_t = a$. Substituting for this in equation (3) and integrating gives:

$$\ln VA_t = \ln B_0 + a\ln w_t + (1 - a)\ln r_t + a\ln L_t + (1 - a)\ln K_t \quad (4)$$

where $\ln B_0$ is the constant of integration.

If we further assume that wage and profit rates grow at constant rates, i.e., $\varphi_{wt} = \varphi_w$ and $\varphi_{rt} = \varphi_r$ (we shall relax this assumption below), then substituting into equation (4) and taking anti-logarithms gives:

$$VA_t = B_0 e^{\varphi t} L_t^a K_t^{(1-a)} \quad (5)$$

where t is a linear time trend. It should be noted that equation (5) is mathematically equivalent to the specification of the Cobb–Douglas production function with constant returns to scale, where the exponents a and $(1-a)$ are the output elasticities, and the rate of exogenous technical progress is given by $\varphi = a\varphi_w + (1 - a)\varphi_r$. (If we assume the 'stylized fact' that the rate of profit does not vary over time, then $\varphi = a\varphi_w$.)

The important point to note is that equation (5) is not a Cobb–Douglas production function and, in fact, well-defined micro-production functions may not even exist. It is simply the identity (2) under some assumptions that do not necessarily have anything to do with the existence of an aggregate production function.[1] For example, factor shares may be constant because firms pursue a constant mark-up pricing policy.

The above implies that if the assumptions made (i.e., factor shares together with the weighted growth rate of the wages and the rate of profits are constant) were correct, and we were to estimate the form $VA_t = A_0 e^{\lambda t} L_t^{\alpha} K_t^{\beta}$, where λ is the putative rate of 'technical progress', as:

$$\ln VA_t = \ln A_0 + \lambda t + \alpha \ln L_t + \beta \ln K_t + u_t \quad (6)$$

where u_t is the error term, we should expect a perfect fit with the estimates of α and β equalling the factor shares, and the estimate of λ, the rate of TFP growth, equal to φ, the weighted average of the growth rates of the wage and profit rates.[2] Because of these results, the erroneous conclusion could be drawn that factors are paid their marginal products and constant returns to scale prevail.

[1] Notice that the issue being discussed here is very different from the 'path dependency' problem discussed by Hulten (1973). Hulten was addressing the question of whether integration of the Solow residual would necessarily get back to the shift parameter $A(t)$ in the production function. In our argument, there need be no production function. Furthermore, while Hulten proved that if there is no production function, the Solow residual cannot be line-integrated to a unique solution, this is fundamentally different from what is being discussed here.

[2] Our argument also applies to the estimation of frontier production functions (e.g., Mahadevan and Kalirajan, 2000). This approach tries to decompose TFP growth into improvements in technical efficiency (embodied technological change) and technical progress *per se* (the shift in the production function).

Any divergence from a perfect statistical fit will be simply due to the fact that either factor shares are not constant or the variation in $a\ln w_t+(1-a)\ln r_t$ is not fully explained by a linear time trend, or both. The failure to explain completely the variation in $a\ln w_t+(1-a)\ln r_t$ is akin to misspecification due to omitted-variable bias. If the trend does not proxy correctly the two variables omitted from the identity, namely, $\ln w_t$ and $\ln r_t$, the parameters of $\ln L_t$ and $\ln K_t$ are likely to be biased, and hence diverge from the factor shares. It is only if $\ln w_t$ and $\ln r_t$ are orthogonal to $\ln L_t$ and $\ln K_t$, that the parameters of the latter will give estimates equal to the values of the factor shares.[1] In practice, it is normally the inappropriateness of using a *linear* time trend, rather than variation in the factor shares, that gives a poor fit to the supposed production function.

It is worth emphasising what this argument implies. The income accounting identity is, of course, compatible with any production technology or, indeed, with the lack of a production function, but the data given by the identity will always give a perfect fit to a putative Cobb–Douglas production function so long as factor shares are constant. There are many reasons why factor shares are constant, such as a constant mark-up on unit labour or variable costs, as we have already noted, or the Kaldorian theory of distribution (Kaldor, 1955–6), neither of which depends upon the existence of an underlying Cobb–Douglas production function. But if this is the case, the data will give a good fit to the Cobb–Douglas relationship even if the true underlying technology was, for example, fixed coefficients (see Shaikh, 1987).[2]

2.2 A generalisation to any production function: the Equifinality Theorem

The argument is more general than that discussed above for the Cobb–Douglas production function (as we shall see in Section 3 below, when we consider specifically the translog production function), and applies to *any* specification of a 'production function.' Indeed, we may term this result the Equifinality Theorem. It is useful to consider this in three parts.

(i) Let us assume that there exist well-defined individual micro-production functions $Q_i = F_i(L_i, K_i^r, t)$, that are expressed in physical terms. Let us further assume that the aggregate economy *cannot* be represented as a well-defined technological relationship between aggregate output and aggregate inputs. This is because the stringent conditions for aggregation are not satisfied (Fisher, 1969, 1993) or there are other problems (Harcourt, 1972) and so an aggregate production function does not exist. Nevertheless, the individual firms' accounting identities can be summed arithmetically to give equation (2) for the whole economy, or for some sub-sector such as manufacturing. This identity can always be transformed into a mathematical expression of the form $VA = F(L, K, t)$ that will give a good statistical fit to the data under appropriate assumptions about the time paths of factor shares and φ_t (the weighted growth of the real wage and the rate of profit). The estimated function $F(\bullet)$ will have all the apparent properties of a well-behaved neoclassical aggregate production function, such as constant returns to scale, and diminishing returns to each

[1] For expositional ease, we ignore the econometric problems posed by the possible non-stationarity of the variables. This issue does not undermine the generality of the argument. Note that, since we are dealing with an accounting identity, they must be of secondary importance. Felipe and Holz (2001) deal with this issue.

[2] Hsing (1992) argues that an error by Solow (1957) in the calculation of discrete growth rates means that his specification of the production function is not consistent with the neutrality and homogeneity postulate. Hsing argues that when this is corrected, theoretically the best specification is given by a linear accounting identity with the coefficients of L and K representing the initial period wage and rental rates. However, McCombie (1996) has shown that Hsing's argument is not correct and the inferences he draws cannot be substantiated. Hsing's analysis, however, does not affect the argument concerning the accounting identity in this paper.

factor; and it will also satisfy the marginal productivity conditions for factor pricing as the estimated output elasticities will equal the observed factor shares. However, this estimation is not capturing a technological relationship.

The reason for this is that equation (2), by virtue of being an identity, must hold whatever the underlying technological relationships of the economy.

We may make this argument in a slightly different way. If we erroneously assume that the economy can be represented by an aggregate production function, then $VA = F(L, K, t)$ may be expressed in growth rate form as:

$$va_t = \frac{\partial VA_t/\partial t}{VA_t} + \left(\frac{\partial VA_t}{\partial L_t} \cdot \frac{L_t}{VA_t}\right)l_t + \left(\frac{\partial VA_t}{\partial K_t} \cdot \frac{K_t}{VA_t}\right)k_t = \lambda_t + \alpha_t l_t + \beta_t k_t \quad (7)$$

If it is assumed that there is perfect competition and factors are paid their marginal products, it follows that $\alpha_t = a_t$ and $\beta_t = (1-a_t)$, and equation (7) may be written as $va_t = \lambda_t + a_t l_t + (1 - a_t)k_t$. But this is formally equivalent to equation (3), where $\lambda_t = \varphi_t$. Moreover, as equation (3) must always hold, so conversely it should be possible from this to find a specific functional form for $VA = F(L, K, t)$, even though no aggregate production function actually exists.

In his oft-cited simulation experiments, Fisher (1971) specified a number of micro Cobb–Douglas production functions which he aggregated and which explicitly violated the aggregation conditions. To his surprise, and contrary to his expectations, it gave a good statistical fit to the data. He concluded: 'The point of our results, however, is not that an aggregate Cobb–Douglas fails to work well when labor's share ceases to be roughly constant; it is that an aggregate Cobb–Douglas will continue to work well so long as labor's share continues to be roughly constant, even though that rough constancy is not itself a consequence of the economy having a technology that is truly summarized by an aggregate Cobb–Douglas' (Fisher, 1971, p. 307). The reason for this is simply that the factor shares were approximately constant at the macro level (see Shaikh, 1980).

It should be noted that, even if the aggregation conditions are met, in the sense that an aggregate production function *does* exist, the underlying identity will still ensure that the estimated output elasticities will always equal the factor shares, regardless of the state of competition. Even in this case, the parameters cannot necessarily be interpreted as reflecting the underlying technology as the values of, and variation in, the factor shares could be due to, say, socio-economic factors (such as changes in the relative bargaining power of trade unions and capital).

The problem is that value added is a value measure and not a physical quantity, notwithstanding the fact that constant-price value added (or gross output) is, as we have noted above, often referred to as the 'quantity' or 'volume' of output. Of course, if we were in a corn, or leets, world there should be no theoretical problem in estimating a production function. This is also true when we deal with engineering production functions. In these cases, the (micro) production function is $Q = F(L, K', t)$ and the accounting identity is $pQ = wL + rK'$, where Q is a physical quantity, r here is the price of the capital good, and p is the price (in, say, £s) per unit of Q. Estimation of such production function would yield the technological parameters, provided that we can accurately proxy the exact path of technical progress, for the reasons set out above.[1]

[1] This is reminiscent of the problem of how to determine the shape of the production function, and whether it is possible to separate shifts from movements along it. On this see Diamond *et al.*'s (1972) Impossibility Theorem. While it is true that is an issue when using physical quantities, it is not when we deal with values, as we will always get a good statistical fit to a supposed production function.

(ii) Let us assume that the production relations at the micro level are *not* characterised by any well-defined micro-production functions (and, by inference, there is no aggregate production function). The use of value data in the estimation will still enable an 'aggregate production function' to give a good statistical fit. In this case, the accounting identity ensures that there will be a good statistical fit to the input–output data at the aggregate level. This, once again, cannot be interpreted as evidence that the aggregate production function exists, and the estimated coefficients cannot be interpreted as necessarily reflecting technological parameters.

(iii) From equation (3), we know that $\varphi_t \equiv a_t\varphi_{wt} + (1-a_t)\varphi_{rt}$ and that this equation is, by definition, equivalent to the Solow residual, $\lambda_t \equiv va_t - a_t l_t - (1-a_t)k_t$. As the expression for φ_t given by equation (3) can also be obtained from a neoclassical cost function and the use of the usual neoclassical assumptions, $\varphi_t \equiv a_t\varphi_{wt} + (1-a_t)\varphi_{rt}$ is regarded in the neoclassical framework as the factor-price measure of technical progress (Young, 1998; Hsieh, 1999).[1] However, the accounting identity poses the same problem for the dual as it does for the primal. To demonstrate this, we shall briefly discuss the neoclassical methodology in deriving the rate of 'technical change' from the cost function (an explicit discussion of which may be found in Whiteman, 1988) and then show how the same results may be simply obtained from the accounting identity.

Consider first a production function with factor-augmenting technical change, namely $Q = F(A_L L, A_K K)$, where A_L and A_K are indices of labour and capital efficiency, and the rates of labour-augmenting and capital-augmenting technical progress are given by λ_L and λ_K. Duality theory shows that the production function has a cost function of the form $C^* = f(\tilde{w}^*, \tilde{r}^*, Q)$, where C^* is total nominal costs and $\tilde{w}^*$ and $\tilde{r}^*$ are the efficiency adjusted or effective nominal wage and profit rates, i.e., w^*/A_L and r^*/A_K respectively.[2] (The asterisk denotes a nominal value.) Assuming constant returns to scale, the cost function can be expressed as $c^* = g(\tilde{w}^*, \tilde{r}^*) = g(w^*/A_L, r^*/A_K)$, where c^* is the unit cost of production. This may take an explicit functional form such as the Cobb–Douglas or the translog cost function.[3] Whiteman (1988) uses the translog unit cost function, and differentiating this with respect to factor prices and using Sheppard's lemma gives an equation for each of the factor shares. Moreover, partially differentiating the unit cost function with respect to time and substituting in the equations for factor shares gives an expression for the dual rate of cost diminution as:

$$\frac{\partial \ln c^*}{\partial t} = -\lambda_t = -[a_t\lambda_{Lt} + (1 - a_t)\lambda_{Kt}] \qquad (8)$$

[1] Hsieh (1999) uses the dual approach and finds the growth of TFP for East Asia to be much higher than Young's original estimates. Young (1998) in a critique of Hsieh's earlier working papers argues that the differences are due to computational and methodological errors.

[2] Strictly speaking, r^* is the nominal user cost of capital. This, however, does not seriously affect the argument. Value added is calculated summing physical output using market prices. If product markets are not competitive, the resulting measure of value added will contain an element due to monopoly profits. It could be argued that a correct measure of constant price output should exclude this component and should be calculated using marginal costs. In this case the argument concerning the identity follows through exactly as, assuming labour markets are competitive, $\Sigma x_i Q_i \equiv wL + r_c K$, where x is the marginal cost, and r_c is the rental price or user cost of capital. When conventional value added is used, the rate of return implicit in the identity is the rate of profit calculated from the national accounts and, indeed, this is how it is derived in many growth accounting exercises (see, for example, Harberger, 1988, p. 29).

[3] It should be noted that 'the translog cost function is not derived from any optimising procedure in the face of the translog production function. The translog has no self-dual' (Heathfield and Wibe, 1987). This does not affect our argument, as the critique applies equally to neoclassical cost functions, *per se*.

If we totally differentiate the cost function with respect to time and equate the rate of change in product prices ($\hat{p}$) with the rate of change in unit costs ($\hat{c}^\star$) we obtain an expression for the rate of technical change as:

$$\lambda_t = a_t\varphi^\star_{wt} + (1 - a_t)\varphi^\star_{rt} - \hat{p}_t = a_t\varphi_{wt} + (1 - a_t)\varphi_{rt} \tag{9}$$

Thus, the conclusion from this neoclassical analysis is that the rate of technical change is measured as the difference between the weighted sum of the growth of nominal wages and the rate of profit and the rate of change in prices. This is the dual approach to deriving (and estimating) the rate of technical change.

An alternative neoclassical approach is to start initially from the underlying identity. Harberger (1998) provides an example of this methodology (see also Jorgenson and Griliches, 1967). The rate of 'real cost reduction' is defined by Harberger as $R \equiv w_0\Delta L + (\rho_0 + \delta_0)\Delta K - P\Delta VA$, where the subscript 0 denotes the initial value, ρ is the rate of return on capital, δ is the rate of depreciation, VA is GDP, and P is its price index. Ignoring the interaction term, it can be seen that this can be expressed in the traditional growth rate form as $\varphi_t = R/PVA = a\,\Delta L/L + (1-a)\Delta K/K - \Delta VA/VA$, where a and $(1-a)$ are the factor shares. In Harberger's (1998) analysis, however, he assumes that each factor is rewarded according to its marginal product. Hence, underlying the identity is a neoclassical production function or a cost function and the usual optimising assumptions.[1] The growth of the residual may be written as what Harberger terms a 'dual representation', and this takes the form $\varphi_t = a(\Delta w^\star/w^\star) + (1-a)(\Delta r^\star/r^\star) - \Delta P/P$, where $r_0{}^\star$ denotes the nominal value of $(\rho_0 + \delta_0)$. This, in discrete growth form, is equal to $\hat{c}^\star$. However, the results of both these approaches (i.e., starting from either the cost function or the 'neoclassical' identity *à la* Harberger) may be equally derived either from the accounting identity without the need for any assumptions of an underlying neoclassical cost function or a production function or both, and without the need to use duality theory.

Let us consider the accounting identity for value added in nominal terms:

$$VA^\star_t = w^\star_t L_t + r^\star_t K_t \tag{10}$$

where the asterisk again denotes nominal values. In growth rates:

$$va^\star_t = a_t\varphi^\star_{wt} + (1 - a_t)\varphi^\star_{rt} + a_t l_t + (1 - a_t)k_t \tag{11}$$

The identity in constant prices was given by equation (2), and in growth rates by equation (3). Thus, it follows from the above equations that the growth of the implicit price deflator ($\hat{P}_t$) equals:

$$va^\star_t - va_t = \hat{P}_t = a_t(\varphi^\star_{wt} - \varphi_{wt}) + (1 - a_t)(\varphi^\star_{rt} - \varphi_{rt}) \tag{12}$$

and the rate of cost diminution, or the rate of change in unit costs of production, is given by:

$$\hat{c}^\star_t = \hat{P}_t - [a_t\varphi^\star_{wt} + (1 - a_t)\varphi^\star_{rt}] = -[a_t\varphi_{wt} + (1 - a_t)\varphi_{rt}] = -\varphi_t \tag{13}$$

Given the above, i.e., the fact that we have derived φ_t simply as a transformation of the income accounting identities for value added in nominal and real terms, the interpretation of the Solow residual as a measure of technical change does not necessarily follow.

[1] There is a contradiction in Harberger's analysis, since he argues that the residual could be due to changes in x-efficiency such as when 'real costs were reduced when a very lax manager was replaced by someone more strict' (Harberger, 1998, p. 3). This is inconsistent with the neoclassical optimising assumptions that imply firms are technically efficient.

Furthermore, it is misleading to refer to $\varphi_t = a_t\varphi_{wt} + (1 - a_t)\varphi_{rt} = \lambda_t$ as the dual, or price-based measure of technical progress (the cost reduction interpretation of productivity growth), of $\varphi_t = va_t - a_t l_t - (1 - a_t)k_t$.

What about the argument that an economy that experiences an increase in both its real wage and rate of profit must have increased its overall level of productivity or efficiency? It could be argued that φ_t measures such a rate of growth of efficiency. Certainly, under these circumstances, the economy is in a Pareto preferred situation, but the point to note is that it is not possible to ascribe this unambiguously to a result of technical change. There is no reason to assume that the factor shares equal the output elasticities of the true aggregate production function (if it, in fact, exists) or that production is necessarily subject to constant returns to scale (although this is what the use of value data will show). The point to note is that our derivation of what we have referred to above as rate of 'cost diminution' is simply a tautology derived from an identity with no behavioural assumptions.

Wages are likely to be correlated with labour productivity (the latter, unlike the notion of Solow residual, does not require any underlying theoretical assumptions about the existence of a production function or the state of competition). Changes in the rate of profit are also likely to be associated with changes in the capital–output ratio. But the only possible way to argue that φ_t is a measure of the rate of technical change (or of the negative of the rate of real cost reduction, in the sense used by Harberger) is to postulate the existence of an aggregate production function or cost function, together with the conditions for producer equilibrium. This is required as a justification for using the factor shares to weight φ_w and φ_r to derived a combined index of TFP growth and to justify regarding $a_t l$ and $(1-a_t)k$ as a measure of the contribution, in a causal sense, of the growth of the factor inputs to output growth.

As the Solow residual may be merely derived from the accounting identity, the growth of an input multiplied by its factor share does not necessarily measure that input's contribution to the growth of output in any causal sense. Hence, it is not possible to ascribe φ_t uniquely to technical change, while the ability to do this, of course, is the whole rationale for the growth accounting approach.[1]

The growth accounting approach must, of necessity, *assume* that there is a well-behaved underlying production function, and that the usual neoclassical assumptions (including a competitive equilibrium) are fulfilled. There is a 'Catch-22' problem here. One can assume the existence of an aggregate production function and try to estimate such parameters as the elasticity of substitution; but it is not possible to test that this is a technological parameter (because of the arguments in the Equifinality Theorem). Similarly, one can use the growth accounting methodology to quantify the 'residual', $\lambda_t = va_t - a_t l_t - (1 - a_t)k_t$, which will equal $\varphi_t = a_t\varphi_{wt} + (1 - a_t)\varphi_{rt}$, but this will in no way imply that that what has been calculated is (unambiguously) the rate of technical progress; and neither will it provide any independent support for the appropriateness of the methodology used. The equation $\varphi_t = a_t\varphi_{wt} + (1 - a_t)\varphi_{rt}$ can be interpreted unambiguously only as an arbitrarily weighted combination of the wage and profit rates and, as such, as a measure of distributional change, although not in a zero-sum sense.

[1] The reason that the accounting identity has not been seen to pose any problem in the neoclassical analysis is possibly that at the aggregate level output is assumed to be analogous to a (physical) quantity index, Q. The accounting identity is assumed in the theoretical model to be, as we noted above, $pQ = wL + rK'$, from which it is always possible to derive or recover Q and to specify the production function as $Q = F(L, K', t)$ or the cost function as $C = f(w, r, Q, t)$.

The implications of the Equifinality Theorem may be summarised as follows.

(a) It is not possible to test statistically, and hence potentially refute, the hypothesis that the economy is representable by an aggregate production function.
(b) The values of the factor shares determine the values of the output elasticities in a statistical sense, rather than the other way around, for economic reasons (such as factors are paid their marginal products under competitive conditions).
(c) The first-order conditions, which may be erroneously interpreted as derived from the marginal productivity theory of factor pricing, will always be statistically significant, and will not be rejected by the data. It is consequently not possible to test this theory statistically (this is elaborated below).
(d) The functional form must always be homogeneous of degree one in L and K, and its econometric estimation must yield an R^2 of unity, provided a close approximation to the linear accounting identity has been found.[1]
(e) The statistical results of estimating a particular specification of $VA = F(L, K, t)$ should only be interpreted as a test for how accurately the paths of the factor shares and φ_t are being tracked. For example, estimating a Cobb–Douglas with a linear time trend can only be regarded as a test of the null hypothesis that factor shares, and the weighted average of the growth rates of the wage rate and the profit rate are constant over time.
(f) Any statistical contradiction of (a) to (e) is merely the result of a poor approximation to the identity. It will always be possible to find a better approximation for the identity given by equation (2) where such refutations do not occur.[2]

3. The study of Kim and Lau (1994)

Kim and Lau (1994) used a more general production function than the Cobb–Douglas, namely, the translog, to explain the growth of the high-performing economies of East Asia, and (supposedly) to test the hypothesis of constant returns to scale, and the marginal productivity theory of factor pricing under competitive conditions (see also Boskin and Lau, 1990).

Kim and Lau used a meta-production function. This is defined as the common underlying production function that can be used to represent the input–output relationship of a given industry in all countries. Essentially, it amounts to pooling time-series and cross-section data. This, according to the authors, has the advantage that since inter-country data normally show greater variability than time-series data for a single country, the parameters of the production function can be estimated with greater precision. Likewise, this specification, according to Kim and Lau, does not depend upon the assumptions of constant returns to scale, neutral technical progress and profit maximisation with competitive output and input markets—assumptions that underlie most growth accounting exercises. These hypotheses, and the postulate that all countries share the same production

[1] This follows from the fact that what is being estimated is simply an identity. However, paradoxically, it is easy to show that if the data followed exactly the theoretical paths (e.g., for the Cobb–Douglas constant factor shares and constant growth rates of the wage and profit rate), then perfect multicollinearity would prevent the estimation of the coefficients.

[2] The fact that, in practice, estimations of 'production functions' sometimes provide poor statistical fits and implausible values for the estimated parameters may be responsible for the mistaken belief that a behavioural (and hence refutable) relationship is being estimated. For example, suppose that factor shares are roughly constant and the conventional Cobb–Douglas, with a linear time trend, is estimated. Although $a\ln w_t + (1 - a)\ln r_t$ may show a strong secular trend, the presence of cyclical fluctuations in this trend may be sufficient to render the estimated coefficients seriously biased.

function, are supposedly capable of direct testing, in marked contrast to the growth accounting approach. For empirical purposes, they used the translog production function and allowed for biased technical change. In other words, the contribution of capital- and labour-augmenting technical change were estimated separately.[1]

Pooling time series and cross-section data for the G-5 countries and for the four East Asian NIEs, they found TFP growth was not significantly different from zero in the latter, and that technical progress was capital augmenting. The explanation for this finding, they argued, is that, until very recently, the NIEs had not invested in R&D, and that most technical progress was embodied in the capital goods (therefore, exogenous technical progress had to be negligible). They also conjectured that the 'software' component of investments, i.e., managerial methods, institutional environment, as well as supporting infrastructure, lags behind the 'hardware' component. Likewise, they rejected statistically the standard assumptions behind growth accounting, i.e., an aggregate production function homogeneous of degree one in capital and labour, neutral technical progress, and profit maximisation. Since we have argued above that, because of the underlying identity, it should not be possible to reject these putative hypotheses, it is worthwhile considering their approach in greater detail to see the reasons for this.

The translog production function with the inputs measured in terms of efficiency units is given by:

$$\ln VA_t = \ln A_0 + \alpha \ln A_{Lt}L_t + \beta \ln A_{Kt}K_t + \gamma(\ln A_{Lt}L_t \ln A_{Kt}K_t) + \delta(\ln A_{Lt}L_t)^2 + \varepsilon(\ln A_{Kt}K_t)^2 \tag{14}$$

where A_{Lt} and A_{Kt} are the levels of factor-augmenting technology, termed the augmentation level parameters, which are allowed to differ between countries, and where λ_L and λ_K are the rates of labour and capital-augmenting technical change. Substituting $\ln A_{Lt} = \ln A_{L0} + \lambda_L t$ and $\ln A_{Kt} = \ln A_{K0} + \lambda_K t$, where A_{L0} and A_{K0} are the initial levels, the following estimating equation is derived:

$$\begin{aligned} \ln VA_t = {} & \ln A_0 + \delta(\ln A_{L0})^2 + \varepsilon(\ln A_{K0})^2 + \gamma(\ln A_{L0})(\ln A_{K0}) + \alpha \ln A_{L0} + \beta \ln A_{K0} \\ & + (\alpha + \gamma \ln A_{K0} + 2\delta \ln A_{L0})\ln L_t + (\beta + \gamma \ln A_{L0} + 2\varepsilon \ln A_{K0})\ln K_t \\ & + \delta(\ln L_t)^2 + \varepsilon(\ln K_t)^2 + \gamma(\ln L_t)(\ln K_t) \\ & + (2\varepsilon\lambda_K + \gamma\lambda_L)(t\ln K_t) + (2\delta\lambda_L + \gamma\lambda_K)(t\ln L_t) \\ & + [(\beta + \gamma \ln A_{L0} + 2\varepsilon \ln A_{K0})\lambda_K + (\alpha + \gamma \ln A_{K0} + 2\delta \ln A_{L0})\lambda_L]t \\ & + [\varepsilon(\lambda_K)^2 + \delta(\lambda_L)^2 + \gamma\lambda_L\lambda_K]t^2 \end{aligned} \tag{15}$$

or, equivalently:

$$\begin{aligned} \ln VA_t = {} & c + b_1 \ln L_t + b_2 \ln K_t + b_3(\ln K_t)^2 + b_4(\ln L_t)^2 + b_5(\ln L_t \ln K_t) \\ & + b_6(t\ln K_t) + b_7(t\ln L_t) + b_8 t + b_9 t^2 \end{aligned} \tag{16}$$

Equation (16) was estimated by Kim and Lau in first differences together with the corresponding first-order condition for labour, i.e., a system of two equations reflecting the technical relations and the economic decisions of the firm. The reason is that it is inappropriate to estimate the production function as a single regression equation treating capital and labour as exogenous variables. Consequently, as A_{L0} and A_{K0} differ between

[1] Kim and Lau (1994) also included a term to capture the exogenous improvement in output efficiency (quality). This is found to be statistically insignificant and, for expositional ease, we have not included it in our discussion, and it does not materially affect the argument.

countries, the coefficients c, b_1, b_2 and b_8 are country-specific constants. Estimates of the rate of factor augmenting technical change may be obtained using $\hat{\varepsilon}$, $\hat{\delta}$, $\hat{\gamma}$, and the coefficients of $(t\ln K)$ and $(t\ln L)$ or their first differences if the growth rate specification is used. The coefficient of t^2 is not independent, but is determined by $\hat{\varepsilon}$, $\hat{\delta}$, $\hat{\gamma}$, $\hat{\lambda}_L$ and $\hat{\lambda}_K$.

Expressions for the output elasticities may be obtained by differentiating $\ln VA_t$ with respect to $\ln L_t$ and $\ln K_t$ respectively. If profit maximisation and perfect competition hold, these will equal the relevant factor shares:

$$\frac{\partial \ln VA}{\partial \ln L_t} = (\alpha + \gamma\ln A_{K0} + 2\delta\ln A_{L0}) + (2\delta\lambda_L + \gamma\lambda_K)t + 2\delta\ln L_t + \gamma\ln K_t = a_t \quad (17)$$

and

$$\frac{\partial \ln VA}{\partial \ln K_t} = (\beta + \gamma\ln A_{L0} + 2\varepsilon\ln A_{K0}) + (2\varepsilon\lambda_K + \gamma\lambda_L)t + 2\varepsilon\ln K_t + \gamma\ln L_t = (1 - a_t) \quad (18)$$

Thus, the test of the assumption of a competitive labour market is to determine whether or not the coefficients in (17) are statistically significantly different from those in equation (15). If they are, the argument is that this is sufficient to reject the null hypothesis. Alternatively, we could test the hypothesis that the output elasticities equal the factor shares.

However, in the light of our earlier comments, it will come as no surprise to learn that these arguments are invalidated by the underlying identity, as will now be shown. (See also McCombie and Dixon, 1991.) To commence, let us differentiate equation (16), the 'production function', with respect to time. Denoting growth rates by the lower case, we obtain:

$$va_t = \alpha_t'\lambda_L + \beta_t'\lambda_K + \alpha_t' l_t + \beta_t' k_t \quad (19)$$

where

$$\alpha_\tau' = (\alpha + \gamma\ln A_{K0} + 2\delta\ln A_{L0}) + (2\delta\lambda_L + \gamma\lambda_K)t + 2\delta\ln L_t + \gamma\ln K_t \quad (20)$$

and

$$\beta_t' = (\beta + \gamma\ln A_{L0} + 2\varepsilon\ln A_{K0}) + (2\varepsilon\lambda_K + \gamma\lambda_L)t + 2\varepsilon\ln K_t + \gamma\ln L_t \quad (21)$$

where the variables α_t' and β_t' are the respective output elasticities. From the marginal productivity conditions, equations (17) and (18), $\alpha_t' = a_t$ and $\beta_t' = (1 - a_t)$. Thus, if there is profit maximisation and perfect competition, then from equations (17)–(21) the following relationship must hold:

$$va_t = a_t\lambda_L + (1 - a_t)\lambda_K + a_t l_t + (1 - a_t)k_t = \lambda_t + a_t l_t + (1 + a_t)k_t \quad (22)$$

where a_t and $(1-a_t)$ are labour's and capital's factor shares.

But recall that differentiating the identity (2) with respect to time, we obtained equation (3). Since the latter is obtained from an identity, it must *always* hold for any putative production function, and does not involve the assumption that factors are paid their marginal products.

Comparing equations (22) and (3), it can be seen that (22) will always hold by virtue of the underlying identity. Consequently, *labour's and capital's output elasticities must equal their respective factor shares, regardless of whether or not markets are competitive.* It is thus not possible to test this hypothesis by the procedure adopted by Kim and Lau. Moreover, constant returns to scale must prevail, as from equations (20)–(22), $\beta_t' = (1 - \alpha_t')$. Thus doubling the growth rates of labour and capital will double the growth rate of output.

The translog form can be derived from the identity in a manner analogous to that for the Cobb–Douglas as follows. Assume that φ_t follows a linear time trend (as in the case of the Cobb–Douglas) or some more complex function of time, and that factor shares in this economy happen to be well tracked empirically by the expressions in equations (20) and (21). (This is comparable to the assumption of constant factor shares to derive the Cobb–Douglas.) If these equations are substituted into the identity (3) and the equation is integrated, the translog is derived.

How then that Kim and Lau claimed to have refuted the relationships derived from the identity? In answering this question, let us, for expositional ease, consider the Cobb–Douglas production function and the supposed testing of the marginal productivity conditions. The Cobb–Douglas production function with a time trend was expressed in logarithmic form in equation (6). The following relationships may be obtained from the first-order conditions, also expressed in logarithms:

$$\ln w_t = \ln\alpha + \ln(VA_t/L_t) \tag{23}$$

and

$$\ln r_t = \ln\beta + \ln(VA_t/K_t) \tag{24}$$

where α and β are labour's and capital's elasticities, respectively. Thus, a putative test of the marginal productivity conditions would be to estimate the system of equations:

$$\ln VA_t = b_{10} + b_{11}t + b_{12}\ln L_t + b_{13}\ln K_t + u_{1t} \tag{25}$$

$$\ln w_t = b_{14} + b_{15}\ln(VA_t/L_t) + u_{2t} \tag{26}$$

$$\ln r_t = b_{16} + b_{17}\ln(VA_t/K_t) + u_{3t} \tag{27}$$

and to determine whether $\hat{b}_{12}$ differed from $\exp(\hat{b}_{14})$ and $\hat{b}_{13}$ differed from $\exp(\hat{b}_{16})$. Moreover, $\hat{b}_{15}$ and $\hat{b}_{17}$ should not be statistically significantly different from unity (an alternative approach would be to test whether the estimates of $\hat{b}_{12}$ and $\hat{b}_{13}$ differed significantly from their average factor shares. However, the method we are considering here has certain expositional advantages).

The problem with these supposed tests of the hypothesis of profit maximisation is that, if factor shares are constant (for any of the reasons discussed in Section 2), the null hypothesis cannot be refuted, solely by virtue of the underlying identity. This is because it follows from the definition of the labour share, $w_tL_t/VA_t = a$ (a constant), that $\ln w_t = \ln a + \ln(VA_t/L_t)$, which is formally equivalent to equation (26). The exception, as we have mentioned above, would be when $(a\ln w_t + (1-a)\ln r_t)$ could not be adequately proxied by a linear time trend, so that equation (25) does not provide a good approximation to the identity. In this case, $\hat{b}_{12} \neq \exp(\hat{b}_{14})$. But this should not be taken as a refutation of the marginal productivity conditions, which are, it is argued, untestable. We can always find a better approximation to $a\ln w_t + (1-a)\ln r_t$, so that $\hat{b}_{12}$ will converge to $\exp(\hat{b}_{14})$. For example, if it is found that $a\ln w_t + (1-a)\ln r_t$ varies procyclically around a trend, the inclusion of any procyclical variable, such as capacity utilisation, may well improve the approximation of the identity. It should be noted that the capacity utilisation variable is merely included because it varies procyclically and not for any economic reason although, in production function studies, it is argued that this will provide a necessary adjustment to the flow of services of the factor inputs (see McCombie, 2000–2001).

The limitation with the Cobb–Douglas is that if shares vary too much over time, we may not obtain a very good fit. The advantage of the translog is that by making factor shares a function of l, k, λ_L and λ_K, it allows for a better approximation (a more flexible functional form) than the Cobb–Douglas to the accounting identity. However, we should not necessarily expect the parameters ε, γ and δ to be very stable over time, since the values they take are merely coincidental, and do not necessarily reflect any underlying technology.[1]

Let us return to Kim and Lau's supposed refutation of the conditions of profit maximisation and constant returns to scale. Their procedure is identical to that outlined above with respect to the Cobb–Douglas production function, except that they use a translog production function, and the marginal productivity relationships are expressed in terms of factor shares. Any failure to confirm the marginal productivity conditions arises simply because Kim and Lau do not have a sufficiently close approximation to the accounting identity. In other words, if the specification were to be improved so that the R^2, for example, tends to unity, then we know that these 'hypotheses' cannot be rejected. There are a number of reasons why it is possible in practice for these hypotheses to be rejected, which were discussed above. If the fit is not perfect, it simply means that the translog is not the right approximation to the accounting identity.

This may be made clearer as follows. It will be recalled that the identity in growth rate form, equation (3), is given by $va_t = a_t\varphi_{wt} + (1 - a_t)\varphi_{rt} + a_t l_t + (1 + a_t)k_t$. The factor shares have time subscripts because they vary over time and, if this variation is sufficiently large, as we have noted, it may be sufficient to give a poor fit to the Cobb–Douglas production function. Consequently, we need a better approximation that tracks this variation of the shares. We can see why the translog gives a better fit than the Cobb–Douglas production function since, from equation (20), labour's share is given by $a_t = f(\ln L_t, \ln K_t, t)$ where $\ln L_t$ and $\ln K_t$ are also likely to be trended. Thus, even though there may be no technological relationship between a_t and $\ln L_t$ and $\ln K_t$ through the parameters of a production function, we may get a better fit with the translog compared with the Cobb–Douglas, because the latter makes some allowance for factor shares varying. Of course, if the variation in factor shares is random, any improvement in the fit may be coincidental, and we still may not get a very good fit. It is perhaps worth emphasising again that given the empirical degree of variation in factor shares, the poor fit is normally the result of the inappropriate assumption of a smooth rate of technical change—namely, the weighted average of the growth of the wage rate and the rate of profit—rather than the failure to track the factor shares accurately. Empirically, estimating the full identity given by equations (3) or (4) normally produces highly significant estimates of the factor shares. It is only when $a\ln w_t + (1-a)\ln r_t$ is replaced by a linear time trend that the other coefficients become either statistically insignificant or implausible.

Consequently, Kim and Lau's (1994) claim that their formulation does not depend on the traditional assumptions of growth accounting is ill founded. The fact that they rejected constant returns to scale and profit maximisation was due to the fact that they pooled data for nine countries, and the translog with constant rates of factor augmenting technical change may not be the correct approximation to the identity, i.e., it is a misspecified approximation. The conclusion is that Kim and Lau's two-equation model cannot be used to test the assumptions underlying the aggregate production function. It is nothing

[1] The Cobb–Douglas is a special case of the translog (when the restriction $\gamma = \delta = \varepsilon = 0$ is imposed). As factor shares are constant, however, it is not possible to derive separately the estimates of the degree of labour and capital-augmenting technical change.

more than an approximation to the identity, and as we improve the goodness of fit (by, for example, introducing a more complex time trend, including, for example, sines and cosines) to model $a_t \ln w_t + (1 - a_t)\ln r_t$ more closely, the identity tells that us we shall be increasingly unable to reject the 'hypothesis' of constant returns to scale, and the marginal productivity theory of factor pricing.

4. Empirical evidence

This section provides empirical evidence in order to illustrate the theoretical arguments in the previous sections, using data for Singapore for 1970–1990.[1] To begin with, we estimated the standard translog and Cobb–Douglas functions, including, in both cases, a linear time trend (denoted by t), which supposedly captures the rate of technical progress (as in the orthodox analysis). (The specification of the translog is a more simple function than that used by Kim and Lau, but this does not affect the substance of our argument.) The results are shown in Table 1. It is worth mentioning that the Cobb–Douglas restriction of the translog form, as well as the null hypothesis of constant returns to scale, are rejected. Likewise, in the Cobb–Douglas case, constant returns are rejected (in favour of decreasing returns), and the labour and capital elasticities are significantly different from the average factor shares.[2] The results of these tests would therefore seem to indicate that the aggregate production function of Singapore shows the properties of varying returns to scale and factor elasticities.

Table 2 reports the estimates of equation (3). This equation, as discussed above, was derived as a transformation of the accounting identity by assuming only that factor shares

Table 1. *OLS estimates of the translog and the Cobb–Douglas production functions, for Singapore, 1970–90*

Translog Constant	t	$\ln L_t$	$\ln K_t$	$(\ln L_t)^2$	$(\ln K_t)^2$	$\ln K_t \ln L_t$
−0·320 (−3·50)	0·024 (3·19)	1·146 (6·44)	0·079 (0·83)	−5·398 (−2·73)	−0·697 (−3·65)	4·036 (3·36)
$R^2 = 0{\cdot}999$; $DW = 1{\cdot}47$; Cobb–Douglas restrictions: $F(3, 14) = 13{\cdot}29$ (p value = 0·000)						
H_0: Constant returns to scale, $\chi^2_3 = 35{\cdot}86$ (p value = 0·000)						

Cobb–Douglas Constant	t	$\ln L_t$	$\ln K_t$
−0·500 (−7·86)	0·040 (8·04)	0·607 (5·00)	0·102 (1·80)
$R^2 = 0{\cdot}997$; $DW = 0{\cdot}87$			
H_0: Constant returns to scale, $\chi^2_3 = 8{\cdot}82$ (p value = 0·003)			

Note: t statistics in parentheses.

[1] We are grateful to Chen Kang for providing us with the complete data set. See Chen (1991).

[2] Interestingly, if a quadratic trend is added to the regression, the results indicate significant increasing returns to scale of 1.29. The two factor elasticities continue being significantly different from the factor shares.

Table 2. *OLS estimates of the Cobb–Douglas identity for Singapore*

Levels Constant	$\ln w_t$	$\ln r_t$	$\ln L_t$	$\ln K_t$
−0·140 (−10·17)	0·387 (45·26)	0·585 (51·60)	0·354 (18·02)	0·613 (68·57)
$R^2 = 0{\cdot}9999$; $DW = 1{\cdot}56$				
Growth rates	φ_{wt}	φ_{rt}	l_t	k_t
	0·384 (17·28)	0·584 (20·78)	0·370 (8·18)	0·610 (30·02)
$R^2 = 0{\cdot}985$; $DW = 2{\cdot}32$				
Approximation to the identity $A(t)$	$\ln L_t$	$\ln K_t$		
0·010 (6·25)	0·389 (3·15)	0·553 (13·50)		
$R^2 = 0{\cdot}9967$; $DW = 1{\cdot}25$				
H_0: Elasticity of labour (η_L) = 0·39, χ_1^2 = 0·0000013 (p value = 1·000)				
H_0: Elasticity of capital (η_K) = 0·61, χ_1^2 = 1·88 (p value = 0·170)				
H_0: Constant returns to scale, χ_1^2 = 0·45 (p value = 0·500)				

Notes: t statistics in parentheses.
The average labour share for 1970–90, $\bar{a}$, takes a value of 0·39, with a maximum value of 0·42 and a minimum value of 0·32· The standard deviation of labour's share is 0·039.

are constant. The equation is estimated in unrestricted form, and the purpose is to test empirically whether, for this data set, the assumption of constant factor shares is reasonable. The results indicate that indeed this is the case. As may be seen from Table 2, the four estimates of the factor shares are very close to the corresponding values of the actual average factor shares. When the specification is in terms of levels, the estimates of the labour share are 0·387 and 0·354, while in terms of growth rates they are 0·384 and 0·370· This compares with an average value over the period of 0·39. The extremely high t values and close statistical fit, together with the proximity of the estimates to the factor shares in both the regressions, demonstrate a very good approximation to the income accounting identity. All the results of the Equifinality Theorem are confirmed by these regressions.

If these results are compared with those in Table 1, the contrast in the values of the estimated parameters of labour and capital is particularly evident. The time trend (and the higher order terms in the translog) in these regressions act as a proxy variable for the weighted average of the logarithms of the wage and profit rates. The results reported in Table 1, however, are not optimal. While the statistical fit is very high, the substantial change in the estimates of parameters in the Cobb–Douglas, compared with the relevant ones in Table 2, indicates that the approximation is not sufficiently close. We also calculated the factor elasticities implied by the translog. They are given by ($\partial \ln VA/\partial \ln L$) for labour, and ($\partial \ln VA/\partial \ln K$) for capital and the results are reported in Table 3 (η_L is the elasticity of labour, η_K is the elasticity of capital, a is labour's share, and $1 - a$ is capital's share). The negative elasticities obtained in six cases, and the substantially low returns to

Table 3. *Implied output elasticities of the translog production function for Singapore*

Elasticity of labour: $\eta_L = 1.1462 - 2 \times 5.3975 \ln L_t + 4.0363 \ln K_t$
Elasticity of capital: $\eta_K = 0.0786 - 2 \times 0.6968 \ln K_t + 4.0363 \ln L_t$

Year	η_L	a	η_K	$1-a$	$\eta_L + \eta_K$
1970	−0·4955	0·4041	0·5083	0·5958	0·0127
1971	−0·1471	0·3963	0·4067	0·6036	0·2596
1972	0·2370	0·3605	0·2889	0·6394	0·5260
1973	0·1524	0·3429	0·3411	0·6571	0·4936
1974	0·5091	0·3201	0·2264	0·6798	0·7355
1975	0·9008	0·3522	0·0950	0·6477	0·9956
1976	0·8967	0·3510	0·1085	0·6489	1·0052
1977	0·6745	0·3545	0·2001	0·6454	0·8746
1978	0·3266	0·3731	0·3415	0·6269	0·6682
1979	−0·1556	0·4110	0·5330	0·5889	0·3774
1980	−0·1852	0·3816	0·5605	0·6183	0·3753
1981	0·0444	0·3924	0·4891	0·6075	0·5335
1982	0·4016	0·4256	0·3723	0·5744	0·7738
1983	0·5415	0·4366	0·3326	0·5634	0·8741
1984	0·8479	0·4475	0·2286	0·5524	1·0766
1985	1·5928	0·4801	−0·0408	0·5198	1·5520
1986	1·7357	0·4464	−0·0882	0·5536	1·6475
1987	1·4559	0·4109	0·0227	0·5891	1·4786
1988	1·2695	0·3905	0·0993	0·6095	1·3688
1989	1·0109	0·4000	0·2049	0·6000	1·2158
1990	0·6883	0·4011	0·3324	0·5989	1·0208

scale found for some years, indicate a serious misspecification if this is interpreted as a production function.[1]

An implication of the discussion in the previous sections is that in general the income accounting identity can be approximated by a suitable function of the form $VA = F(L, K, t)$. The problem is to find the correct approximation to the weighted average of the logarithm of the wage and profit rate as a function of time. There is no alternative to finding such an approximation other than by a trial-and-error process. The closer the specification approaches the identity, the closer the parameters will be to the factor shares. By this method, we ended up estimating the last equation in Table 2, where $A(t) = (\text{constant} + \sin(t) + \sin(t^2) - \cos(t^2) + \sin(t^3) + \cos(t^3))^2$ and where sin denotes the sine function, cos is the cosine function, and t denotes the time trend. There is no *a priori* reason, and nothing in neoclassical production theory to suggest that the growth rate of TFP should follow a linear trend with a random error. From the relative constancy of the factor shares reported in the upper part of Table 2, we know that the third equation in Table 2 is virtually the identity. The term $A(t)$ consequently closely approximates the expression $a\ln w_t + (1 - a)\ln r_t$, where $a = 0{\cdot}39$ and $(1 - a) = 0{\cdot}61$ are the average factor shares. The correlation between $0{\cdot}39\ln w_t + 0{\cdot}61\ln r_t$ and $A(t)$ is 0·814.

The important aspect of the third equation in Table 2 is that it takes us back to the identity (although the approximation is not quite perfect). This can be seen by the fact that the estimated coefficients are much closer than before to the factor shares. The null

[1] This results confirm Hulten's view: 'Estimation of the translog (or other flexible) function can lead to parameter estimates that imply oddly shaped isoquants, causing practitioners to place a priori restrictions on the values of these parameters' (Hulten, 2000, pp. 22–3).

hypotheses that the estimated elasticities η_L and η_K equal the average factor shares cannot be rejected, and we obtain 'constant returns to scale', which the theoretical derivation indicates should be the case. This is, of course, precisely what is to be expected, as the specification of the putative production function is now a close approximation to the accounting identity.[1] (See Felipe, 2001.)

Finally, it is also clear that the first-order conditions cannot be statistically rejected. Under the assumptions of profit maximisation and competitive markets, factor prices are determined by their marginal products. This analysis, which is strictly speaking applicable to the microeconomic level, has been extended to the macro level. The first-order condition for labour in this model states that the wage rate equals the marginal product of labour, i.e., $w = p(\partial Q/\partial L)$. However, at the aggregate level the measure of output is value added, and consequently, from the accounting identity given by equation (2), it also follows that $w = \partial VA/\partial L$. The question arises as to how this can be formulated as a testable proposition.

The arguments in Sections 2 and 3 indicate that if the translog were the best approximation to the identity, one would not be able to reject statistically the first-order conditions, namely the relations given by equations (20) and (21). However, it has been seen that, in fact, factor shares were (approximately) constant in Singapore during the period under consideration.[2] This implies that the Cobb–Douglas first-order conditions should not be rejected, as indeed is the case. The marginal productivity conditions in the case of the Cobb–Douglas production function are $w = \alpha(VA/L)$ and $r = \beta(VA/K)$, where α, β, are the output elasticities of labour and capital (which equal factor shares when markets are perfectly competitive, and this is the null hypothesis to test). The OLS regression of the wage and profit rate on labour and capital productivity, respectively, gives $\hat{\alpha} = 0.40$ (with a t statistic of 48·65) for the labour first-order condition; and $\hat{\beta} = 0.61$ (77·38) for capital's first-order condition. Both elasticities are very close to the their respective average factor shares.

Hence, these regressions show how it is always possible, with sufficient ingenuity, to obtain a near perfect statistical fit to the 'aggregate production function' with the supposed output elasticities equalling the factor shares, regardless of the state of competition. This is because all that it is being estimated is an approximation to an identity and the results can tell us nothing about the underlying technology of the economy.

5. The studies of Young

Unlike Kim and Lau (1994), Young (1992, 1995) assumed profit maximisation and perfect competition, and used the Tornqvist index to estimate the rate of TFP growth in a series of growth accounting exercises for the East Asian NIEs. The Tornqvist index is used to take account of the fact that growth rates are calculated over discrete, and not instantaneous, time periods. Diewert (1976) proved that this index is an exact measure of technical change if the underlying production function exhibits constant returns to scale and is

[1] Interestingly, if the coefficient of $A(t)$, 1.0% per annum, were interpreted as the rate of Singapore's TFP growth, this estimate would be slightly above the results reported in the literature for the city-state. On the other hand, the rate of growth of TFP calculated through growth accounting indicates a residual of −0.006% per annum.

[2] This indeed proves to be the case. The estimation of equation (17). The result is: $a_t = 0.33 + 0.005\,t + 0.031\ln L_t - 0.025\ln K_t$ (with t values of 3.32, 0.69, 0.16 and −0.29, respectively). The null hypothesis that the time trend and the logs of labour and capital are jointly equal to zero cannot be rejected. This indicates that the labour share is better approximated by a constant, as in the Cobb–Douglas form.

a translog. Under these circumstances, one has to use the average factor shares of periods t and $(t - 1)$ and calculate the growth rates of output and factor inputs as the difference of the logarithms in periods t and $(t - 1)$. This method, although it involves some rather restrictive assumptions, avoids the problems derived from the econometric estimation of production functions such as the possibilities of multicollinearity and spuriousness of the regressions. Young (1992), in his controversial study for Singapore and Hong Kong, concluded that TFP growth had, on average, been low, if not zero, in Singapore over the last 30 years. In contrast, in the case of Hong Kong, it accounted for up to 30% of overall growth. Young (1995) extended the exercise to include South Korea and Taiwan. These two also had positive TFP growth rates, but nothing exceptional compared with other countries.

The work of Young has been criticised, in particular his results for Singapore, on several grounds. Pack and Page (1994B) and Nelson and Pack (1999) have commented that the output elasticities of capital used by Young for Singapore, over the various subperiods, are very high, around 0·5. Krugman (1992) pointed out that, in Singapore, being an open economy, measures of real output are essentially measures of real value added. However, these are possibly biased by serious problems of quality adjustment. There is also the problem that if either the supply of labour and capital are not perfectly elastic at the aggregate level, or imperfect competition prevails, or both, then even under neoclassical assumptions, the use of factor shares as weights is not the appropriate procedure (Dhrymes; 1965; Brown, 1966; Hall, 1988). Stiglitz (1997, p. 16) recently asked: 'Does anyone who has studied wage setting in Singapore, for example, really believe that wages are set in a competitive process, so that the real wage equals the marginal product of labour, as most of the studies assume?' Surprisingly, however, Young seems content to accept the assumption that markets in the developing countries are perfectly competitive.[1]

These critiques, though they may well be correct, are not so fundamental as the issues that have been have raised above. Even if Young had estimated a positive and high TFP growth rate for Singapore, as in the case of Hong Kong, the arguments advanced in this paper would be unaffected. The issue at hand is the meaning of such an exercise. All Young did was, in effect, to calculate the weighted average of the growth rates of the wage rate and of the profit rate, and concluded that it was zero. This, from the Equifinality Theorem, cannot necessarily be taken to be a measure of technical change. Recall that from equation (3) that $\varphi_t = a_t\varphi_{wt} + (1 - a_t)\varphi_{rt}$. Therefore, one can argue that if $\varphi_t = 0$, then in the case of Singapore $a_t\varphi_{wt} = -(1 - a_t)\varphi_{rt}$. According to Young's data, $a_t \simeq 0.5$.This implies $\varphi_{wt} \simeq -\varphi_{rt}$. It indicates that the growth rate of the wage rate was matched by a decline in the growth rate of the accounting profit rate (from a high initial level). But given that we are dealing with an accounting identity, this says nothing about TFP growth (as understood in the neoclassical sense). This result only indicates that Singapore's profit rate declined during the period analysed. The neoclassical argument is that this decline is due to diminishing returns to capital, consequent upon the rapid rate of capital accumulation. However, it was more likely to be due to the rapid rise in real wages, partly encouraged by Singaporean government policy post 1979 in an attempt to stimulate productivity growth. (It may be recalled that from the identity the profit rate is $r_t=\Pi_t/K_t=(VA_t-w_tL_t)/K_t$ (i.e., total profits, Π, divided by the value of the stock of capital). This has nothing necessarily to do with a

[1] Young (1995, p. 648) was well aware of this problem, and his paper contains a discussion of the possible biases introduced by incorrectly assuming constant returns and perfect competition. He dismissed it as merely a minor problem rather too quickly. For an attempt to construct a measure of TFP growth that does not require the assumption of perfect competition, see Wan (1995). However, it can be shown that his measure may simply be derived from the manipulation of the accounting identity (Felipe and McCombie, 1999).

production function and with the notion of TFP growth as a measure of technical progress.[1] (See Felipe, 2000.)

Young (1994B) extended his approach to use regression analysis. It is sometimes conceded that, because estimations of the production function are often on the basis of single equation models, there may be identification problems (Griliches and Mairesse, 1998). However, see Zellner *et al.* (1966) for a model that justifies the single equation estimation of the production function. Nevertheless, we have seen why the single equation model, if correctly specified, will always give a good statistical fit and elasticities will be equal to the factor shares. Young's (1994B) regression analysis was simpler than that of Kim and Lau (1994). Young used Summers and Heston's (1991) data set to estimate a cross-country production function using 118 countries for the period 1970–85. The perpetual inventory method, with the assumption of a 6% depreciation rate, was used to obtain fairly crude estimates of the capital stock. Regressing the growth of output per worker on a constant and the growth of the capital–labour ratio $(k_i–l_i)$ gave the result:

$$(q_i - l_i) = -0{\cdot}21 + 0{\cdot}45(k_i - l_i) + \mu_i \tag{28}$$

The residual μ_i measures the growth of country *i*'s TFP less the world average (no statistical diagnostics were reported). Young notes that the residuals for the East Asian countries are very close in value to his much more detailed analysis using the growth accounting methodology. Singapore, for example, has an annual growth rate of TFP of –0·4% per annum from the growth accounting exercise, whereas the value obtained from the regression is 0·1% per annum. But what does the regression tell us? We know that from the accounting identity

$$\begin{aligned}(q_i - l_i) &= a_i\varphi_{wi} + (1 - a_i)\varphi_{ri} + (1 - a_i)(k_i - l_i)\\ &= \varphi_i + (1 - a_i)(k_i - l_i)\end{aligned} \tag{29}$$

where the subscript *i* denotes the *i*th country.[2] Consequently, if we estimate $(q_i - l_i) = b_{18} + b_{19}(k_i - l_i) + \mu_i$, it is apparent that the estimate of b_{19} will be the average value of the share of capital, and the sum of the constant (b_{18}) and μ_i will, by definition, provide an estimate of φ_i [the estimates may be subject to some bias if φ_i is not orthogonal to $(k_i - l_i)$]. Thus, it is hardly surprising that the estimates from this 'back-of-the envelope' calculation do not differ from the more detailed studies. The fact that the estimated slope coefficient is reasonably close to the average factor share has no implications for whether or not perfect competition is a reasonable first approximation for analysing the growth rates of these countries.

6. Conclusions

It has been argued in this paper that the recent literature on the sources of growth in East Asia is problematic. The reason is that, as any empirical analysis using the aggregate

[1] Hsieh (1999) has estimated the dual or price-based measure of TFP growth using this framework. Proceeding as we did in Section 3, he derives equation (3) and claims that φ_t provides a measure of TFP growth. Furthermore, he even stresses the advantage that the derivation needs no assumptions. This argument is incorrect. Equation (3) is simply an identity and does require the usual neoclassical assumptions for φ_t to be interpreted as a measure of TFP growth.

[2] Simon and Levy (1963) showed the relationship between the Cobb–Douglas and the accounting identity for cross-section data. The derivation, using a Taylor series expansion, assumes that wages and profit rates in the value-added identity refer to averages for the cross-section. The same result can be derived assuming constant factor shares across sectors (McCombie, 1987).

production function must use value data as opposed to physical quantities, the income identity that relates output (value added) to the sum of the wage bill plus overall profits can be rewritten as a form that resembles a production function. The implication for applied work is that if the estimated functional form is the correct one given the data set, the estimated coefficients must be the factor shares in output. Likewise, the Solow residual is, by definition, a weighted average of the growth rates of the wage and profit rates, the latter being calculated from the national accounts. These conclusions are true always by virtue of the accounting identity, and hold for all economic units, regardless of the state of competition. These results have been formalised in the Equifinality Theorem. As, in practice, nearly all empirical work on production functions uses aggregated data (for either sectors or the whole economy), these must be constant price value data. This, however, leaves the notion of the aggregate production function in a difficult position for the empirical analyses of growth and technical progress.

The literature on the sources of growth in East Asia provides an excellent example of the theoretical problems discussed in this paper. We conclude that the methodological problems underlying the conventional neoclassical approach are serious enough to question the relevance of the whole discussion of whether TFP growth in East Asia is zero, positive or negative. This is irrespective of whether one uses growth accounting or estimates econometrically the 'aggregate production function'. (See also Kaldor (1957) and Pasinetti (1959) for alternative critiques of the neoclassical approach.) In a sense, this paper has been somewhat nihilistic. An implication, though, is that a more fruitful way of understanding why East Asia grew so rapidly is to be found at the micro level, especially within a framework where the creation and/or assimilation of technology is explicitly discussed, and using firm-level data (see, for example, Hobday, 1995). This suggestion was advanced long ago by Nelson (1973, 1981) but, unfortunately, has been almost totally ignored. Nevertheless, a pioneering attempt along these lines has been the 'matched samples' study of Mason *et al.* (1996), who looked at the reasons for the productivity differences between British, German, French and Dutch firms (directly at the plant level). This, we suggest, is likely to give more insights into the growth process than further replications of the growth accounting exercise or the fitting of aggregate production functions.

Bibliography

Bernanke, B. 1987. Comment, *NBER, Macroeconomics Annual*, vol. 2, 202–5

Boskin, M. J. and Lau, L. J. 1990. 'Post-war economic growth in the Group-of-five-countries: A new analysis', Technical Paper No. 217, Stanford, CA, Center for Economic Policy Research, Stanford University

Brown, M. 1966. *On the Theory and Measurement of Technological Change*, Cambridge, Cambridge University Press

Chen, K. 1991. The form of Singapore's aggregate production function and its policy implications, *Singapore Economic Review*, vol. XXXVI, 81–91

Denison, E. 1962. *The Source of Economic Growth in the United States and the Alternatives Before Us*, New York, Committee for Economic Development

Denison, E. 1967. *Why Growth Rates Differ: Postwar Experience in Nine Western Countries*, Washington, DC, The Brookings Institution

Denison, E. 1985. *Trends in American Economic Growth 1929–82*, Washington, DC, The Brookings Institution

Diamond, P., McFadden, D. and Rodriguez, M. 1972. Identification of the elasticity of substitution and the bias of technical change, in McFadden, D. (ed.), *An Econometric Approach to Production Theory*, Amsterdam, North Holland

Diewert, E. W. 1976. Exact and superlative index numbers, *Journal of Econometrics*, vol. 4, 115–46

Dhrymes, P. J. 1965. Some extensions and tests for the CES class of production function, *Review of Economics and Statistics*, vol. 39, 357–66

Felipe, J. 1999. Total factor productivity growth in East Asia: a critical survey, *Journal of Development Studies*, vol. 35, 1–41

Felipe, J. 2000. On the myth and mystery of Singapore's 'zero TFP', *Asian Economic Journal*, vol. 14, 187–209

Felipe, J. 2001. Endogenous growth, increasing returns, and externalities: an alternative interpretation of the evidence, *Metroeconomica*, vol. 52, 391–427

Felipe, J. and Fisher, F. M. 2003. Aggregation in production functions: what applied economists should know, *Metroeconomica*, vol. 54, 208–62.

Felipe, J. and Holz, C. 2001. Why do aggregate production functions work? Fisher's simulations, Shaikh's identity, and some new results, *International Review of Applied Economics*, vol. 15, 261–85

Felipe, J. and McCombie, J. S. L. 1999. Wan's 'new approach' to technical change: a comment, *Journal of Comparative Economics*, vol. 27, 355–63

Felipe, J. and McCombie, J. S. L. 2001. Biased technical change, growth accounting and the conundrum of the East Asian Miracle, *Journal of Comparative Economics*, vol. 29, 542–65

Ferguson, C. E. 1971. Capital theory up to date: a comment on Mrs. Robinson's article, *Canadian Journal of Economics*, vol. IV, 250–4

Fisher, F. M. 1969. The existence of aggregate production functions, *Econometrica*, vol. 37, 553–77

Fisher, F. M. 1971. Aggregate production functions and the explanation of wages: a simulation experiment, *Review of Economics and Statistics*, vol. LIII, 305–25

Fisher, F. M. 1993. *Aggregation. Aggregate Production Functions and Related Topics*, edited by Monz, J., London, Harvester Wheatsheaf

Friedman, M. 1953. The methodology of positive economics, in Friedman, M. (ed.), *Essays in Positive Economics*, Chicago, University of Chicago Press

Griliches, Z. and Mairesse, J. 1998. Production functions: the search for identification, in Strom, S. (ed.), *Econometrics and Economic Theory in the 20th Century*, Cambridge, Cambridge University Press

Hall, R. E. 1988. The relation between price and marginal cost in U.S. industry, *Journal of Political Economy*, vol. 96, 921–47

Harberger, A. C. 1998. A vision of the growth process, *American Economic Review*, vol. 88, 1–32

Harcourt. G. C. 1972. *Some Cambridge Controversies in the Theory of Capital*, Cambridge, Cambridge University Press

Heathfield, D. F. and Wibe, S. 1987. *An Introduction to Cost and Production Functions*, Basingstoke, Macmillan

Hobday, M. 1995. *Innovation in East Asia. The Challenge to Japan*, Cheltenham, Edward Elgar

Hsieh, C.-T. 1999. Productivity growth and factor prices in Asia, *American Economic Review, Papers and Proceedings*, vol. 89, 133–8

Hsing, M. 1992. On the measurement of aggregate production functions, *Cambridge Journal of Economics*, vol. 16, 463–74

Hulten C. R. 1973. Divisia index numbers, *Econometrica*, vol. 41, 1017–25

Hulten, C. R 2000. 'Total Factor Productivity: A Short Biography', Working Paper No.7471, National Bureau of Economic Research, Cambridge, MA

Jorgenson, D. W. and Griliches, Z. 1967. The explanation of productivity change, *Review of Economic Studies*, vol. 34, 249–83

Jorgenson, D. W., Gollop, F. and Fraumeni, B. 1987. *Productivity and U.S. Economic Growth*, Cambridge, MA, Harvard University Press

Kaldor, N. 1955–6. Alternative theories of distribution, *Review of Economic Studies*, vol. 23, 83–100

Kaldor, N. 1957. A model of economic growth, *Economic Journal*, vol. 67, 591–624

Kim, J.-I. and Lau, L. 1994. The sources of economic growth of the East Asian newly industrialised countries, *Journal of the Japanese and International Economies*, vol. 8, 235–71

Krugman, P. 1992. Comment on 'A tale of two cities: factor accumulation and technical change in Hong-Kong and Singapore', *NBER, Macroeconomics Annual*, vol. 7, 54–6

Krugman, P. 1994. The myth of Asia's miracle, *Foreign Affairs*, November/December, 62–78

Mahadevan, R., and Kalirajan, K. P. 2000. Singapore's manufacturing's TFP growth: a decomposition analysis, *Journal of Comparative Economics*, vol. 28, 828–39

Mason, G, van Ark, B., and Wagner, K. 1996. Workforce skills, product quality and economic performance, pp. 177–97 in Booth, A. L. and Snower, D. J. (eds), *Acquiring Skills. Market Failures, their Symptoms and Policy Responses,* Cambridge, Cambridge University Press

May, K. 1947. Technological change and aggregation, *Econometrica,* vol. 15, 51–63

McCombie, J. S. L. 1987. Does the aggregate production function imply anything about the laws of production? A note on the Simon and Shaikh critiques, *Applied Economics,* vol. 19, 1121–36

McCombie, J. S. L. 1996. On Hsing's critique of Solow's 'Technical change and the aggregate production function', *Cambridge Journal of Economics,* vol. 20, 785–95

McCombie, J. S. L. 1998. Paradigms, rhetoric, and the relevance of the aggregate production function, in Arestis, P. (ed.), *Method, Theory and Policy in Keynes. Essays in Honour of Paul Davidson,* vol. III, Aldershot, Edward Elgar

McCombie, J. S. L. 2000–2001. The Solow residual, technical change and aggregate production functions, *Journal of Post Keynesian Economics,* vol. 23, 267–97 (Errata, vol. 23(3), p. 544)

McCombie, J. S. L. 2001. What do aggregate production functions show? Second thoughts on Solow's 'Second thoughts on growth theory', *Journal of Post Keynesian Economics,* vol. 24, 589–615

McCombie, J. S. L. and Dixon, R. J. 1991. Estimating technical change in aggregate production functions: a critique, *International Review of Applied Economics,* vol. 5. 24–46

Nelson, R. R. 1973. Recent exercises in growth accounting: new understanding or dead end?, *American Economic Review,* vol. 63, 462–8

Nelson, R. R. 1981. Research on productivity growth and productivity differences: dead ends and new departures, *Journal of Economic Literature,* vol. XIX, 1029–64

Nelson, R. R. and Pack, H. 1999. The Asian growth miracle and modern growth theory, *Economic Journal,* vol. 109, 416–36

Pack, H. and Page, J. M. 1994A. Reply to Alwyn Young, *Carnegie-Rochester Conference Series on Public Policy,* vol. 40, 251–7

Pack, H. and Page, J. M. 1994B. Accumulation, exports and growth in the high performing Asian economies, *Carnegie-Rochester Conference Series on Public Policy,* vol. 40, 199–236

Page, J. M. 1994. The East Asian Miracle: four lessons for development policy, *NBER Macroeconomics Annual,* vol. 9, 216–69

Pasinetti, L. L. 1959. On concepts and measures of changes in productivity, *Review of Economics and Statistics,* vol. 41, 270–86

Phelps Brown, E. H. 1957. The meaning of the fitted Cobb–Douglas function, *Quarterly Journal of Economics,* vol. 71, 546–60

Robinson, J. 1953–54. The production function and the theory of capital, *Review of Economic Studies,* vol. 21, 81–106

Robinson, J. 1971. The measure of capital: the end of the controversy, *Economic Journal,* vol. 81, 597–602

Rodrigo, C. G. 2000. East Asia's growth: technology or accumulation?, *Contemporary Economic Policy,* vol. 18, 215–27

Samuelson, P. A. 1979. Paul Douglas's measurement of production functions and marginal productivities, *Journal of Political Economy,* vol. 87, 923–39

Shaikh, A. 1974. Laws of production and laws of algebra: the Humbug production function, *Review of Economics and Statistics,* vol. LVI, 115–20

Shaikh, A. 1980. Laws of production and laws of algebra: Humbug II, in Nell, E. J. (ed.), *Growth, Profits and Property, Essays in the Revival of Political Economy,* Cambridge, Cambridge University Press

Shaikh, A. 1987. Humbug production function, in Eatwell, J., Milgate, M. and Newman, P. (eds), *The New Palgrave. A Dictionary of Economic Theory and Doctrine,* London, Macmillan

Simon, H. A. 1979A. On parsimonious explanation of production relations, *Scandinavian Journal of Economics,* vol. 81 459–74

Simon, H. A. 1979B. Rational decision making in business organizations, *American Economic Review,* vol. 69, 493–513

Simon, H. A. and Levy F. K. 1963. A note on the Cobb–Douglas function, *Review of Economic Studies,* vol. 30, 93–4

Solow, R. M. 1957. Technical change and the aggregate production function, *Review of Economics and Statistics,* vol. 39, 312–20

Solow, R. M. 1974. Law on production and laws of algebra: the Humbug production function: a comment, *Review of Economics and Statistics*, vol. LVI, p. 121

Solow, R. M. 1987, Second thoughts on growth theory, in Steinherr, A. and Weiserbs, D. (eds), *Employment and Growth: Issues for the 1980s*, Dordrecht, Martinus Nijhoff

Stiglitz, J. 1997. The role of government in economic development, in Bruno, M. and Pleskovi, B. (eds), *Annual World Bank Conference on Development Economics 1996*, Washington, DC, World Bank

Summers, R. and Heston, A. 1991. The Penn World table (mark 5): an expanded set of international comparisons, 1950–1988, *Quarterly Journal of Economics*, vol. 106, 327–68

Wan, G. H. 1995. Technical change in Chinese state industry: a new approach, *Journal of Comparative Economics*, vol. 21, 308–25

Whiteman, J. L. 1988. The efficiency of labour and capital in Australian manufacturing, *Applied Economics*, vol. 20, 243–61

Wibe, S. 1984. Engineering production functions: a survey, *Economica*, vol. 51, 401–11

Young, A. 1992. A tale of two cities: factor accumulation and technical change in Hong Kong and Singapore, *NBER Macroeconomics Annual*, vol. 7, 13–63

Young, A. 1994A. Accumulation, exports and growth in the high performing Asian economies. A comment, *Carnegie-Rochester Conference Series on Public Policy*, vol. 40, 237–50

Young, A. 1994B. Lessons from the NICs: a contrarian view, *European Economic Review*, vol. 38 964–73

Young, A. 1995. The tyranny of numbers: confronting the statistical realities of the East Asian growth experience, *Quarterly Journal of Economics*, vol. 110, 641–80

Young, A.1998. 'Alternative Estimates of Productivity Growth in the NIC's: A Comment on the findings of Chang-Tai Hsieh', NBER Working Paper No. 6657

Zellner, A., Kmenta, J. and Drèze, J. 1966. Specification and estimation of Cobb–Douglas production function models, *Econometrica*, vol. 34, 784–95