Are estimates of labour demand functions mere statistical artefacts?

Jesus Felipe [a]; J. S. L. McCombie [b]
[a] Asian Development Bank, Manila, Philippines [b] Cambridge Centre for Economic and Public Policy,
University of Cambridge, UK

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Jesus Felipe[a]* and J.S.L. McCombie[b]

[a]Asian Development Bank, Manila, Philippines; [b]Cambridge Centre for Economic and Public Policy, University of Cambridge, UK

This paper considers the estimation of putative neoclassical aggregate labour demand functions using constant price value data. Regression results normally find that employment is negatively related to the real wage and that the constant-output elasticity of employment with respect to the real wage is about −0.3. This is taken as evidence that unemployment is the result of the real wage being too high, *ceteris paribus*. This paper shows that these estimates are purely the result of an underlying identity and cannot be interpreted as implying any causal relationship and, as such, they have no policy implications.

Keywords: labour demand functions; accounting identity

JEL classification: J23

Introduction

One of the most enduring controversies in macroeconomics is the question as to whether or not unemployment can be largely attributed to the real wage being too high. The question has been interpreted as essentially an empirical issue. The neoclassical approach suggests that, in the long run, capital–labour substitution and wage flexibility guarantee full employment, and, hence, using the neoclassical production function one can derive estimates of the elasticity of employment with respect to the real wage.

Indeed, the motivation for Paul Douglas originally to begin his seminal estimations of the aggregate production function was the spectacle of lecturers in the 1920s drawing labour demand schedules on the blackboard without any idea of the steepness of their (downward) slopes (Douglas 1948). Since the mid-1960s, there have been numerous studies that have attempted to resolve this issue by drawing on neoclassical production theory and explicitly, or implicitly, estimating the elasticity of the demand for labour with respect to the real wage. While a variety of different data sources, estimation techniques and specifications (the modelling of the dynamic adjustments, etc.) has been used, all the studies, in effect, estimate a labour demand function derived from an aggregate production function, although, as we shall see below, the marginal revenue product of labour function has also been used.

The factor supply functions are not normally modelled, as they are assumed to be perfectly elastic for the individual firm. Hamermesh (1993) has provided a useful survey of the literature and although the estimates of the elasticity vary often quite considerably between studies, they are nearly always statistically significant and

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*Corresponding author. Email: jfelipe@adb.org

negative: 'If one were to choose a point estimate for this parameter [the elasticity of labour demand, holding output constant], 0.30 would not be far wrong' (Hamermesh 1993, 92). This is roughly the same figure Douglas (1934) found and is consistent with the Cobb-Douglas production function where labour's share is 0.7. As Hamermesh (1993, 92, omitting a footnote) further remarks, 'the immense literature that estimates the constant-output demand elasticity for labour in the aggregate has truly led us "to arrive where we started and know the place for the first time".'[1]

These results, taken as a whole, have been seen by some as confirming the neoclassical view that an increase in the real wage, *ceteris paribus*, will increase unemployment by lowering the demand for labour.[2] As Lewis and MacDonald (2002, 18) put it: 'The elasticity of demand for labour at the aggregate level is an important parameter for macroeconomic analysis. In particular, policy issues concerning the impact of wage falls on employment hinge on the size of this parameter.' However, we shall show that any policy implications may be very misleading, such as the putative adverse effect on employment of the introduction, or increase, of the minimum wage (Felipe and McCombie 2008). The contention of this paper is that the empirical evidence does not necessarily support the policy conclusions that have been drawn from the various labour demand studies. The problem is that the labour demand function is derived from an aggregate production function. It is now well established that the use of value data (either value added or gross output) poses intractable problems for the interpretation of any statistical estimates derived from the aggregate production function (see, especially, Phelps Brown 1957; Shaikh 1974, 1980; Simon 1979a,b; Felipe and McCombie 2001, 2003b. The issues are surveyed and extended in Felipe and McCombie 2005). It should be noted that we define an aggregate production function as one where constant-price value data are used to measure output and the capital stock. This is in contrast to 'engineering production functions' which use physical data. Hence, under our definition, the aggregate production function can refer to a highly disaggregated specification such as at the four-digit Standard Industrial Classification SIC or even the firm level.

Nevertheless, Michl (1987, 361), for example, has argued that 'the methodology of estimating employment equations does not founder on the shoals of algebraic tautology, which diminish the estimates of some estimates of aggregate production functions, as noted by Shaikh (1974)'. In this paper, we show that this is not the case. Because of an underlying accounting identity, it is possible to obtain a negative value of the elasticity of labour demand with respect to the wage rate, even though there may be no behavioural relationship involved. Indeed, it is very difficult to obtain anything other than a statistically significant negative 'elasticity'. All that is being estimated is an approximation of an identity, which is, of course, true by definition (see also Lavoie 2000).

We commence by briefly setting out the four neoclassical labour demand and marginal revenue functions that have been used to estimate the elasticity of employment with respect to the real wage. We next consider the underlying accounting identity that defines value added, namely that value added is equal to the average wage rate multiplied by the numbers employed plus the rate of profit (or the rental price of capital) multiplied by the capital stock. We show that if factor shares are constant (although our argument does not depend upon this assumption), the logarithm of employment is positively related to the logarithm of output (value added) and negatively related to the logarithm of the real wage rate, the rate of profit and the capital stock. It is shown that it is this underlying identity that generates the negative

relationship between the logarithms of employment and the real wage rate. We demonstrate that the regression equations commonly used to estimate the elasticity of demand for labour are simply a misspecification of the identity (such as through the omission of variables or proxying them by a time trend). Moreover, we further show that if we specify them correctly from a neoclassical point of view by allowing 'technical progress' to be proxied by a non-linear time trend, rather than as a linear time trend as is usually the case, then, as a result, two of the specifications are *exact* identities under the usual neoclassical assumptions.[3] A consequence is that all these estimated regressions have no policy implications.

We further consider the study by Anyadike-Danes and Godley (1989a), which also questions the putative labour demand function. Using a simulation analysis, they show that the logarithm of the real wage rate is statistically significantly inversely related to logarithm of employment, even when it is known by construction that employment is not a function of the real wage. We show that this seemingly perverse result is more generally due to the underlying accounting identity and the arguments that we advance.

The neoclassical theory of the demand for labour

The neoclassical theory underlying the estimation of labour demand functions is now standard. Nevertheless, given the controversy that surrounds certain aspects of the theory (Rowthorn 1999; Dowrick and Wells 2004; Lewis and McDonald 2004), it is useful briefly to rehearse it here.

A well-behaved aggregate production function, $Q = A(t)f(L,K)$, is assumed; perfect competition prevails and factors are paid their marginal products so that from the marginal productivity conditions, $w^n/p = w = f_L$ and $\rho^n/p = \rho = f_K$. Q is the volume of homogeneous output, $A(t)$ is the shift factor, where $A(t) = A_o e^{\lambda t}$ and λ is the exogenous constant rate of technical change. K is the physical capital stock and L the level of employment. p is the price of output. The variables w^n and ρ^n are the nominal wage rate and the rental price of capital, while w and ρ are their values in real (product) terms.

Two different assumptions are made in deriving estimates of two separate types of the elasticity of the demand for labour. One holds output constant, while the other holds capital constant.

Holding output constant

The first assumption is to hold output constant, but to allow the capital–labour ratio to vary as the relative price of the factor inputs changes. Solving the system of equations given by the production function and the two first-order conditions yields a value of the elasticity of demand for labour, namely, $\eta_{LL/Q} = -(1-a)\sigma$, where $(1-a)$ is capital's share in total output (and equals capital's output elasticity $(1-\alpha)$) and σ is the elasticity of substitution. $a\ (=\alpha)$ is labour's share. In the case of a Cobb-Douglas, $\eta_{LL/Q} = -(1-a) \approx -0.30$ given that capital's share of output is approximately 30%. More generally, the elasticity of substitution is generally found by estimating the aggregate production function to be between 0.5 and unity and so we should expect $-0.15 \leq \eta_{LL/Q} \leq -0.30$. This, of course, assumes the existence of an aggregate production function together with an aggregate elasticity of substitution. We shall have more to say on this below. The cross-elasticity of demand for labour is given by $\eta_{LK/Q} = (1-a)\sigma$ and in the case of the Cobb-Douglas this takes a value of approximately 0.30 (Hamermesh 1993, 24).[4]

For the Cobb-Douglas production function, the elasticity of demand for labour is given by:

$$lnL = -lnA_0 + (1-\alpha)ln(\alpha / (1-\alpha)) + lnQ - (1-\alpha)lnw + (1-\alpha)ln\rho - \lambda t \quad (1)$$

where the estimate of the coefficient of *lnw* is the elasticity of demand of labour, as noted above.

Alternatively, the marginal revenue product of labour curve may be derived from the first order conditions by differentiating the Cobb-Douglas production function with respect to labour and equating it to the wage rate (*w*). In logarithmic form this is:

$$lnL = ln\alpha + lnQ - lnw \quad (2)$$

The coefficient of *lnw* (i.e. −1) is not the elasticity of the demand for labour curve, which is equal to $-(1-\alpha)$ and may be derived from the estimate of the intercept.

In fact, equation (2) is not usually estimated, *per se*, but rather it is a restricted case of the more general Constant Elasticity of Substitution (CES) production function, within which it is nested. In the case of this production function, the marginal revenue product of labour curve is given by:

$$lnL = -(1-\sigma)ln\,A_0 + \sigma ln\delta - \sigma ln\,w + lnQ - (1-\sigma)\lambda t \quad (3)$$

It is important to note again that the coefficient of *lnw* is not the elasticity of demand for labour, which, as we noted above, is $\eta_{LL/Q} = -(1-a)\sigma$. Equation (3) is the specification used by Lewis and MacDonald (2002). It can be seen that as σ tends to unity and δ, the distribution parameter of the CES, tends to α, so equation (3) will tend to $lnL = ln\alpha + lnQ - lnw$, which is the Cobb-Douglas specification. It can also be seen that an increase in the real wage, *ceteris paribus*, will result in a decline in employment, which is the crucial result.

Holding the capital stock constant

An alternative assumption, perhaps more suitable for short-run analysis, is to keep the capital stock constant, in which case output varies, but not because of exogenous changes in demand. If the capital stock remains constant, then as employment falls with a rise in the real wage, so output will decrease. Thus, in the CES case the wage elasticity is given by $\eta_{LL/K} = -\sigma/(1-a)$, (Rowthorn 1999), and the expected range of values is $-1.67 \leq \eta_{LL/K} \leq -3.33$ for a value of capital's share of 0.30 and the elasticity of substitution ranging between 0.5 and unity. Thus, the fall in employment is considerably greater when the capital stock cannot alter, compared with when output is constant, which is to be expected.

In the case of the Cobb-Douglas production function, $\eta_{LL/K}$ can be obtained from the first order condition for labour $\frac{\partial Q}{\partial L} = \alpha A(t)\left(\frac{K}{L}\right)^{(1-\alpha)} = w,$ which in logarithmic form becomes:

$$lnL = \frac{1}{(1-\alpha)}ln\,\alpha + \frac{1}{(1-\alpha)}ln\,A_0 + \frac{1}{(1-\alpha)}\lambda t - \frac{1}{(1-\alpha)}ln\,w + ln\,K \qquad (4)$$

where $\eta_{LL/K}$ is the elasticity of demand for labour and this is equal to $-1/(1-\alpha)$ or the negative of the inverse of capital's share (Rowthorn 1999). This is the equation used by Layard, Nickell and Jackman (1991), who estimated equation (4) adding lags. Rowthorn (1999) criticizes Layard et al., on the grounds that the expected elasticity of demand for labour (keeping capital constant) should be around -3, but Layard et al.'s estimates suggest that it is an order of magnitude smaller in the case of many countries, and is often less than unity (Rowthorn 1999, 416, Table 1). The aim of Rowthorn's paper is to show that investment does have some positive effect on employment, *pace* Layard et al., who demonstrate that, using a Cobb-Douglas production function, there is no impact. Rowthorn argues that this result depends upon the elasticity of substitution being unity, whereas he argues Layard et al.'s own estimates show that the elasticity must be substantially below unity.

The problem is that in many of the labour demand studies the distinction between the different specifications of the real wage elasticities (and hence their different *a priori* values) is not made. Indeed, it is often not made explicit whether or not the coefficient of *lnw* should be interpreted as an elasticity of demand. But, for our purposes, this is not an issue – what matters is that all specifications predict a negative coefficient on the logarithmic of the real wage.

A more parsimonious interpretation

For empirical estimation purposes, Q in the previous equations is proxied by value added in constant prices, V, while K is proxied by the value of the capital stock also measured in constant prices, J. The fact that value data are being used implies that we are bound to obtain a close statistical fit to the above equations, purely because of the existence of an underlying accounting identity. This is entirely overlooked in all these studies. The definition of value added is: [5]

$$V \equiv wL + rJ \qquad (5)$$

where r is the rate of profit, rather than the user cost of capital or the rental price of capital. This does not make any difference to the argument as we shall see below.

By differentiating equation (5), expressing the result in growth rates and integrating it, we obtain:

$$V \equiv Bw^a r^{(1-a)} L^a J^{(1-a)} \qquad (6)$$

where B is the constant of integration and equals $a^{-a}(1-a)^{-(1-a)}$, where a and $(1-a)$ are again the shares of labour and capital in total output.

Equation (6) may be written in logarithmic form as:

$$ln\,L \equiv -\frac{1}{a}ln\,B + \frac{1}{a}lnV - ln\,w - \frac{(1-a)}{a}ln\,r - \frac{(1-a)}{a}ln\,J \qquad (7)$$

This equation, which is compatible with any state of competition, any degree of returns to scale and the existence or otherwise of an aggregate production function,

will, of course, give a perfect statistical fit provided that factor shares are constant. There are, in fact, overwhelming theoretical reasons for believing that the aggregate production function does not, in fact, exist (Fisher 1992; Cohen and Harcourt 2003; Felipe and Fisher 2003). The central tenet of our paper is that the various specifications of labour demand functions and marginal revenue curves are nothing more than the identity with one or more of the arguments omitted. (This may, of course, lead to biases on the coefficients of the remaining variables, and sometimes in an unpredictable way.) For example, if we proxy *lnJ* by a linear time trend, we obtain an equation with the same arguments as the labour demand function given by equation (1). The coefficients differ between the two specifications, but this paradox is more apparent than real and is resolved below. It is thus little wonder that the R^2s of these estimated equations are usually high and that there is always a negative relationship between *lnL* and *lnw* (see equation (7)).

It is important to address briefly two points relating to equation (7) and its interpretation. First, we elaborate upon the distinction between the profit rate and the rental price of capital. Secondly, we show that although the neoclassical approach acknowledges the existence of the accounting identity, we are making a very different point.

The neoclassical procedure is to use the rental price of capital, rather than the *ex post* rate of profit, in the identity (see, for example, Hsieh 2002). However, this does not affect the argument: we just have a slightly different specification of the identity. Assuming perfect competition, the rental price of capital is calculated by assuming that the firm maximizes the net present value of its receipts over an infinite time horizon subject to two constraints, namely, the production function and the law of motion of the capital stock. The solution to this problem yields the well-known formula for the rental price of capital $f_k = \rho = \dfrac{q_K}{p}(i + \gamma - \hat{q}_k)$ where i is the rate of discount, p is the price of output, q_K is the price of investment, γ is the rate of depreciation and $\hat{q}_K$ is the degree of revaluation or rate of growth of the capital gain or loss (see Jorgenson and Griliches 1967). Under this approach, the real capital stock is calculated by the UK Office of National Statistics, the OECD and the US Bureau of Labour Statistics (BLS), using the nominal values of the rental price of capital to determine the asset shares in profits with which to weight the various assets (see OECD 2001; Lau and Vaze 2002; BLS 2006). The BLS also adjusts for the rate of corporate taxation in its calculations, and so the rental price of capital can be either gross or net of company taxation. Hsieh (2002) uses the accounting identity within the neoclassical growth accounting framework. He utilizes the rental price gross of corporate taxes because of data limitations, but considers the rental price of capital adjusted for taxation to be the preferable measure of the cost of capital to be used in identity. (Clark and Freeman (1980) also use the rental price of capital net of tax.) To the extent that corporation tax does not differ over the years, the net and gross measures should be closely correlated.

How does the rental price of capital relate to the accounting identity, equation (5)? It is easy to see that equation (5), which does not make any assumption about the state of competition, can be written as:

$$V \equiv wL + rJ \equiv wL + r_c J + r_{nc} J \qquad (8)$$

where r_c and r_{nc} are the competitive and the non-competitive components of the rate of profit, and consequently $r \equiv r_c + r_{nc}$. The rental price of capital (ρ), as noted above,

is calculated under neoclassical assumptions and equals the competitive rate of profit (i.e. $\rho = r_c$).[6] If all markets are competitive, $r_{nc} = 0$ and value added is given by:

$$V \equiv wL + r_cJ \equiv wL + \rho J \tag{9}$$

What about the case where there are abnormal profits and $r_{nc} \neq 0$? In neoclassical production theory, V is used as a proxy for output, but as the latter is a physical measure, it should not include any abnormal profits (as these are a distributional component). The correct identity under neoclassical assumptions when there are abnormal profits should be:[7,8]

$$V' \equiv V - r_{nc}J \equiv wL + r_cJ \equiv wL + \rho J \tag{10}$$

It should be noted that the identity is preserved and consequently, the arguments discussed above follow through when the usual neoclassical assumptions are made, but in terms of the identity given by equation (9).

We have couched the argument in terms of equation (5) in order to stress that the problem of the identity does not require the assumption of perfectly competitive markets (see Felipe and McCombie 2007 for a further discussion). For strict equivalence with the neoclassical approach, r is only equal to ρ if all markets are competitive.[9]

Summing up so far, we could have started our argument with equation (9), using the (competitive) rental price of capital, and then derive an equation equivalent to equation (7), but on the left-hand side we would have had V' instead of V and on right-hand side we would have ρ instead of r.

The second point is that it is true, of course, that the existence of the accounting identity, equation (5), has been known for a long time. Indeed, value added must equal the sum of the compensation to labour and the operating surplus (capital's income or total profits). However, the neoclassical approach assumes that there is an independent underlying aggregate production function that exists (i.e. there are no aggregation problems, etc.) and this can be estimated. *If* perfect competition prevails and *if* factors are paid their marginal products, then Euler's theorem implies that the production function estimates will ensure that there is no adding up problem. Only if these assumptions are met will the estimated output elasticities equal the factor shares. In other words, we are dealing with a behavioural relationship. Our argument is, in fact, as Phelps Brown (1957) pointed out, that the putative aggregate production function and the accounting identity are merely different sides of the same coin. The fact that we often find that the fit is not exact, or the estimated elasticities differ from the values of the factor shares, may be partly the reason that it is often implicitly assumed that the estimation of an aggregate production function yields estimates of a true behavioural relationship.

An alternative interpretation of the equations in the previous section is that all that is being estimated is a pricing equation and the negative coefficient of *lnw* is simply a consequence of this. In these circumstances, the inverse relationship between *lnL* and *lnw*, *ceteris paribus*, has no causal significance whatsoever. To see this, let us assume that the i'th firm pursues a constant mark-up pricing policy on unit labour costs:

$$p_{it} = (1 + \pi_{it})\frac{w_{it}L_{it}}{Q_{it}} \tag{11}$$

where p_{it} is the price of the i'th firm's output at time t measured in, say, £'s per unit output, π_{it} is the mark-up, w_{it}^n is the nominal money wage rate, L_{it} is the number employed and Q_{it} is the output measured in homogeneous units. Let us, for illustrative purposes, further assume that the underlying production function is one with fixed coefficients, so that relative prices have no effect on the choice of the ratio of factor inputs. Total value added is given by $P_t V_{0t} = P_t \Sigma p_{i0} Q_{it} = \Sigma p_{it} Q_{it} = \Sigma (1 + \pi_{it}) w_{it}^n L_{it}$, where P_t is the value-added deflator and equals unity in the base year, V_{0t} is value added at time t measured at base year prices, p_{it} is again the current price per unit of output and p_{i0} is the price in the base year. In practice, the mark-up will be on average direct costs (i.e. including the cost of materials). However, this does not affect the argument.

In this case, it is not possible to recover the physical quantities from the published value data, as individual prices are not generally known to the researcher. This is an important point, as it is often implicitly assumed in applied work that P is analogous to p and so V may be regarded, in practice, as homogeneous physical output. If this were actually the case, then V, L and J would not be definitionally related; the production function could be regarded as a behavioural relationship.

For expositional ease, let us further assume that this equation may be approximated by $P_t V_0 = (1 + \pi) w_{it}^n L_{it}$, where w_t^n is the average nominal wage rate and L_t is aggregate employment. If $(1+\pi)$, the average mark-up, is, say, 1.33, this implies that labour's share will be constant and will take a value of 0.75 as $a = 1/(1+\pi)$.[10] It is worth emphasizing that if the mark-up changes for reasons such as, say, changes in labour's bargaining power, labour's share will change as well. This will be reflected in the estimates of the 'output elasticities' of the supposed aggregate production function. But it should be noted that the causation is from the shares to the 'output elasticities', and not *vice versa*.

For every firm (and every industry) there is an associated revenue (or cost, if there are no economic profits)[11] identity in real terms which, dropping the t subscript for notational convenience is given by: $R_i \equiv w_i L_i + r_i J_i$, where R is total revenue and J is a constant-price *value* measure of the capital stock. Aggregating gives the accounting identity $R \equiv wL + rJ$ (which in turn definitionally equals real value added, V), where w and r are the average real wage and profit rates. This may be expressed in growth rates and then integrated to give as $R \equiv V \equiv Bw^a r^{(1-a)} L^a J^{(1-a)}$, if factor shares are constant. If, as is often empirically the case, w is strongly trended and r is roughly constant, then equation (6) may be expressed as $V = A(t) L^a J^{(1-a)}$, where $A(t) = A_o exp(\lambda t)$ and $\lambda = a\hat{w} + (1 - a)\hat{r} = a\hat{w}$. The reader will appreciate that the rewritten expression (6) is formally equivalent to the Cobb-Douglas but recall that we started with the assumption that the true underlying micro-production functions were ones of fixed coefficients.[12] Nevertheless, if we were estimate $lnV = c_1 + b_1 t + b_2 lnL + b_3 lnJ$, then simply by virtue of the identity, $b_2 = a$ and $b_3 = (1-a)$.

This argument may be easily generalized. Suppose factor shares are not constant over time, but vary, then the Cobb-Douglas will not give the best fit to equation (5). What we require is a more flexible approximation, such as the CES or translog. However, these should not be regarded as 'production functions', *per se*, but simply as mathematical transformations that give a good approximation to equation (5). See Felipe and McCombie (2001, 2003). For example, if shares vary, then the translog 'production function' may give a better approximation to equation (5) in levels than the Cobb-Douglas, but it is nevertheless still merely a functional form that approximates the underlying identity.

Estimating the 'labour demand' function or the accounting identity?

As shown above, we have four equations that are commonly estimated, namely,

$$lnL = -lnA_0 + (1-\alpha)ln[\alpha / (1-\alpha] + lnV - (1-\alpha)lnw + (1-\alpha)lnr - \lambda t \quad (1)$$

$$lnL = ln\alpha + lnV - lnw \quad (2)$$

$$lnL = -(1-\sigma)ln A_0 + \sigma ln\delta + ln V - \sigma lnw - (1-\sigma)\lambda t \quad (3)$$

$$lnL = (1/(1-\alpha))lnA_0 + (1/(1-\alpha)) ln\alpha + lnJ - (1/(1-\alpha))lnw + (1/(1-\alpha))\lambda t \quad (4)$$

Q, as noted above, is proxied by value added in constant prices, V. K is proxied by the value of the capital stock measured in constant prices, J, and, for the moment, r is the *ex post* rate of return and defined as $r \equiv (V - wL)/J$. For the moment we use the rate of profit, rather than the rental price of capital, to emphasize that the argument is not dependent upon any assumptions about the state of competition. In the following analysis r and ρ may be used interchangeably, but recall that the specifications of the identity may differ slightly. See equations (8), (9) and (10).

Clark and Freeman (1980), in their classic study, used equation (1), although they used the rental price of capital instead of the rate of profit, while Lewis and MacDonald (2002) used equation (3), and both studies assume that all markets are competitive. We shall return to these studies below.

Let us consider these four specifications and analyse the conditions under which they become formally equivalent to the identity. The latter, it will be recalled, is given by equation (7). For purposes of the empirical analysis, we shall retain the use of α when discussing the neoclassical interpretation and a when considering the identity.

If we were to estimate the identity, we should expect the estimates to be:

$$ln L \equiv c_2 + 1.33lnV - 1.00lnw - 0.33lnr - 0.33lnJ \quad (12)$$

provided the labour's share, a, is about 0.75.

It should be re-emphasized that as this equation is an identity (as long as factor shares are constant), it is compatible with *any* state of competition and whether or not an aggregate production function exists.

At first sight, as we noted above, it might seem that the identity, given by equations (7) and (12) are incompatible with equation (1), even if we substitute a linear time trend for lnJ as a proxy for it in equation (7). This is because although they contain the same variables, the parameters are different. The coefficients of lnV and lnr in equation (1) are 1.0 and $(1-\alpha)$ respectively, but in the amended equation (7), they are $1/a$ and $-(1-a)/a$ (recalling that, because of the identity, α equals a). However, these discrepancies can be simply reconciled.

Let us start with the identity given by equation (7). First, we know that $lnJ \equiv ln(1-a) + lnV - lnr$ and substituting this into the identity, equation (7), gives $lnL \equiv lna + lnV - lnw$, which is formally equivalent to equation (2). Secondly, it follows from the identity (and the usual neoclassical assumptions)[13] that:

$$ln A(t) \equiv lnA_0 + \lambda(t)t \equiv lnB + alnw + (1-a)lnr \quad (13)$$

where $\lambda(t)$ is not necessarily constant and is likely to be a non-linear function of time with a pronounced cyclical component. It follows that if we equate $-lnA_0 - \lambda(t)t + lnB + alnw + (1-a)lnr = 0$ to $lnL - lna - lnV + lnw = 0$, i.e. to equation (2), and rearrange the terms, we get:

$$ln L \equiv [lnB - lnA_0 + lna] - \lambda(t)t + lnV - (1-a)lnw + (1-a)lnr \qquad (14)$$

where $lnB + lna = (1-a)ln[a/(1-a)]$.

Equation (14) is none other than equation (1), once it is appreciated that $lnA(t)$ is given by equation (13) and it should be recalled that the argument may be couched in terms of either lnr or $ln\rho$. The only difference is that the neoclassical specification derived from the aggregate production function often imposes a *linear* time trend, λt, although there is no theoretical reason for doing this – indeed, quite the opposite (Solow 1957). *In other words, when equation (1), the putative labour demand equation, is correctly specified with a non-linear time trend, it is nothing more than the full identity.* It should be noted that in this analysis we have *not* had to proxy lnJ by a time trend in the identity. We have merely used the identity together with the definition of capital's share and the definition of $A(t)$ to derive the correctly specified (in neoclassical terms) labour demand function. It is correctly specified in that we take the general definition of $A(t)$ rather than arbitrarily assuming that it is proxied by a linear time trend. Equation (1) is only equal to the misspecified identity when it is constrained to have a linear time trend.

The sign of lnr in equation (14) is now positive (compare with equation (7) where it is negative). The former accords with neoclassical production theory in that a rise in the price of capital, *ceteris paribus*, should increase the demand for labour through the factor substitution effect. However, it now can be seen that it is merely a result that *must* always occur because of the identity. It is interesting to note that Clark and Freeman (1980) find the estimate of the coefficient of the logarithm of the price of capital to be positive, even though they use a linear time trend.

In practice, $lnA(t)$ has a distinct procyclical fluctuation, as we shall see, which means that equation (1) may not give a perfect fit to the data, even though factor shares are constant. This may give the misleading impression that we are actually estimating a behavioural equation rather than a (misspecified) identity. It is, however, always possible to derive a non-linear time trend, often including sine and cosines, to give a perfect fit to $lnA(t)$. It also demonstrates that in terms of the neoclassical model, a linear time trend is a poor proxy for the rate of technical progress, if all the usual neoclassical assumptions are met.

If we similarly assume that equation (4) has a non-linear time trend, then under the usual neoclassical assumptions, it may be easily shown that it is an exact identity. Substituting $lnB + alnw + (1-a)lnr \equiv lnA(t)$ and $lnJ \equiv ln(1-a) + lnV - lnr$ into equation (4) gives the identity $lnL \equiv lna + lnV - lnw$.

Thus we have the irony that if we specify the labour demand functions equations (1) and (4) correctly from a neoclassical point of view so that technical change is allowed to vary non-linearly, their estimation will always give a perfect fit as they are merely tracking an identity. However, if technical change is constrained to be a linear function of time, then the labour demand functions are merely tracking a misspecified identity. Equation (2) is definitionally true and equation (3) is simply this identity with a time trend and hence the negative coefficient of lnw will be driven by the identity.

Empirical results

In order to estimate the labour demand functions, we used data for manufacturing over the period 1960–1993 from the NBER Manufacturing Industry Database, supplemented by data from the OECD database.[14]

It is useful to comment on some characteristics of the statistics. The wage rate is strongly trended upwards. Fitting a time trend to *lnw* and estimating it by the Exact AR(1) Newton-Raphson iterative method gives a growth rate of 2.04% per annum (with a t-ratio of 15.40). However, this conceals a cyclical component – the fastest growth of the wage rate in a particular year was 6.50% and the slowest was a decline of 5.38%. The rate of profit showed no well-defined trend, with a statistically insignificant trend growth rate of −0.59% per annum. But the cyclical fluctuations were even more violent than for real wages; the annual growth rates ranged between 13.45% and −23.68% per annum. Consequently, while a linear time trend gives a statistically significant fit to the weighted growth of the wage rate and the rate of profit, it does not capture the cyclical fluctuation and is thus, *ex post*, not a very good proxy. The trend weighted growth of the wage rate and profit rate is 1.53% per annum, with a t-value of 2.27.

We first confirmed empirically the expected results for the coefficients of the identity expressed as equation (7). As we are dealing with an identity, the problem of the endogeneity or otherwise of the regressors does not arise. This also applies to the order of integration of the various variables, which is very much a secondary issue. Consequently, we do not report the usual battery of diagnostic statistics, except for the $\bar{R}^2$, the standard error of the regression (SER) and the Durban Watson diagnostic (DW).

The results of estimating the full identity are reported in Table 1, equation (i).[15] From a consideration of the data, the shares of labour, *a*, are reasonably stable over the long run with a mean of 0.729 and a standard deviation of 0.024. Consequently, it is not surprising that the estimated coefficients are well determined and are close to their expected values, namely 1.338 compared with the theoretical value of 1.371 for *lnV*, −0.996 (−1.000) for *lnw*, −0.349 (−0.372) for *lnr* and −0.348 (−0.372) for *lnJ*.

Equation (ii) reports the results of estimating the identity including the linear time trend, which at this stage can be simply regarded as an irrelevant included variable, and, consequently, its expected coefficient is zero. However, it can be seen that the coefficient is actually statistically significant. We know that this must be purely coincidental, or perhaps occurs because factor shares are not exactly constant and we are consequently estimating the 'wrong' functional form. The other coefficients in the regression are only marginally affected by its inclusion.

Equation (iii) in Table 1 drops the capital stock variable and replaces it with a time trend and the close correspondence of this specification, which has no causal implications, and the labour demand function, equation (1), is readily apparent, apart from the difference in the theoretical values of the coefficients of *hV*, *lnw* and *lnr* (including the sign on the last). The only other difference between this 'labour demand function' and the full identity is that the labour demand function excludes the logarithm of the capital stock and includes a time trend (putatively to capture the growth of total factor productivity). When *lnJ* is simply dropped from the identity, i.e. from equation (i), and replaced by a time trend in equation (iii), it is found that when the AR(2) Newton-Raphson iterative method is used to correct for the autocorrelation, the estimates of the remaining coefficients are scarcely different from those obtained using the full identity. Consequently, we can see that the negative

Table 1.   The 'labour demand function' for US manufacturing, 1960–1993. Dependent Variable: *lnL*.

	$(i)^a$	$(ii)^b$	$(iii)^b$	$(iv)^c$	$(v)^f$	$(vi)^f$
Constant	−0.626	−1.250	−4.045	−1.310	−0.107	1.549
	(−2.32)	(−6.76)	(−13.23)	(−0.77)	(−0.07)	(0.89)
lnV	1.338	1.297	1.271	1.008	0.965	0.948
	(90.60)	(85.47)	(75.29)	(10.89)	(11.03)	(11.13)
lnw	−0.996	−0.960	−0.910	–	−1.013	−0.699
	(−48.27)	(−51.13)	(−35.14)		(−8.40)	(−3.30)
lnr	−0.349	−0.325	−0.307	–	–	–
	(−26.91)	(−42.88)	(−40.13)			
ln(w/r)	–	–	–	−0.129	–	–
				(−4.29)		
lnJ	−0.348	−0.249	–	–	–	–
	(−47.70)	(−12.41)				
t	–	−0.003	−0.012	$−1.332^d$	–	−0.007
		(−5.57)	(−17.39)	(−9.48)		(−1.80)
R^{-2}	0.999	0.999	0.998	0.999	0.935	0.940
SER	0.002	0.002	0.003	0.002	0.016	0.015
DW	1.498	2.445	1.936	0.299^e	1.916	1.812

Notes:
a Exact AR(1) Newton-Raphson iterative method.
b Exact AR(2) Newton-Raphson iterative method.
c OLS, long-run elasticities; one-year lags of *lnL, lnV,* and *lnA*(t)* (or *alnw + (1−a)lnr*).
d Coefficient of *lnA*(t)* (or *alnw + (1−a)lnr*), which is substituted for the linear time trend.
e Durbin's h-test.
f Figures in parentheses are t-values.
g Long-run elasticities; one-year lag of *lnV* and *lnw*. Exact AR(1) Newton-Raphson iterative method.
Sources: NBER Manufacturing Database, OECD database.
Memorandum item: Average share of labour = 0.729.

coefficient of *lnw* in the 'labour demand function' is being driven solely by the underlying accounting identity.

However, as shown above, we can get a perfect correspondence between equation (1) and the identity. In equation (iv), the linear time trend was replaced by $lnA^*(t) \equiv alnw + (1−a)lnr$, which, as we have seen, is what both the neoclassical approach (if all its assumptions are fulfilled) and the identity suggest should be the case. (Equation (iv) and equation (1) are now theoretically identical, provided a flexible time trend is incorporated in the latter.) Because of multicollinearity, we constrained the coefficients of *lnw* and *−lnr* to be the same. Hence, we estimated equation (1) as:

$$lnL = − lnB − lnA^*(t) + (1−\alpha)ln[\alpha / (1−\alpha)] + lnV − (1−\alpha)ln(w / r) \quad (15)$$

where $lnA^*(t) = alnw + (1−a)lnr$.

Equation (iv) reports the OLS long-run estimates when the equation is estimated using one-period lags of *lnL, lnV* and *lnA*(t)*. The estimated coefficient of *lnV* is 1.008 as opposed to its theoretical value of 1.000, the coefficient of *ln(w/r)* is −0.129 compared with −0.271, and of *lnA(t)* is −1.332 compared with −1.000. There is thus a little difference between the estimated values of the coefficients and those that are to be expected from the identity. The disparities are not large and are due to the cyclical

fluctuation in the factor shares. What is important is that the statistically significant negative coefficient of *ln(w/r)* is due to the identity.

These results are similar to those of Clark and Freeman (1980), although they used the rental price of capital instead of the rate of profit and gross output instead of value added. They also obtained a larger value for the coefficient of *ln(w/r)*. As we noted above, they used a linear time trend, which suggests that this was, in fact, a good proxy for the weighted growth of the wage rate and the profit rate.

We also confirmed empirically that using the rental price of capital, as opposed to the rate of profit, did not make any difference to the results reported above. We used the manufacturing capital input estimates (calculated using the implicit rental capital prices, net of taxes, in the aggregation of the assets) calculated by the BLS and we assumed all the usual neoclassical assumptions were met.

The data for manufacturing used by the BLS for calculating estimates of total factor productivity covered the period 1948–2005. Estimating the complete identity by the Exact AR(2) Newton-Raphson Iterative Method gave a very good fit:

$$lnL = -0.930 \qquad +1.490 lnV \qquad -0.986 lnw - 0.500 ln\rho - 0.498 lnJ$$
$$(-66.94) \qquad (155.323) \qquad (-191.58) \quad (-89.07) \quad (-61.67)$$
$$R^2 = 0.9999, \quad SER\ 0.0008, \quad DW = 2.045$$

The average share of labour is 0.664 (with a range from 0.702 to 0.625) and of capital is 0.336 (with a range of 0.375 to 0.298). Consequently, the estimated coefficients are very close to the expected values from the identity, which are 1.50; −1.00; −0.500; and −0.500 respectively.

We also estimated equation (15), i.e. the correctly specified neoclassical labour demand function, using the exact AR(1) Method and we got similar results to those using the *ex post* rate of profit, namely:

$$lnL = -0.641 \qquad -1.069\ lnA^* \quad (t) + 0.939 lnV \quad -0.169 ln(w/\rho)$$
$$(-5.05) \qquad (-18.37) \qquad (33.24) \qquad (-10.0)$$
$$\overline{R}^2 = 0.9993,\ SER = 0.007, \qquad DW = 1.747$$

where *lnA*(t) = alnw + (1−a)lnρ*. The estimated coefficients are close to those of the identity except that the coefficient of *ln(w/ρ)* is about half the expected value of −0.336. When we replaced *lnA*(t)* with a linear time trend, the coefficient of *lnV* was 1.043 (t-value of 5.87). But the coefficient of *ln(w/ρ)*, while −0.125, was statistically insignificant (t-value of −1.48). When we estimated the regression with the coefficients of *lnw* and *lnρ* unconstrained, we found that both were statistically significant but both were also negative. This is what the identity would lead us to expect, but it was not what Clark and Freeman found. They found the coefficient on *lnρ* statistically significant and positive.

The question arises as to whether or not the introduction of lags (based here purely on the criterion of statistical significance) undermines the interpretation of the results as solely reflecting the identity.[16] We know that the identity *lna ≡ lnw + lnL − lnV* holds if *lna* is exactly constant. However, while over the period as a whole, *lna* is

roughly constant, it nevertheless displays a strong cyclical component as evidenced by the following OLS regression:

$$lna = -0.064 + 0.805 \, lna_{-1}$$

$$(-1.77) \quad (7.09)$$

$$\overline{R}^2 = 0.607, \quad SER = 0.020, \quad Durbin \ h–test = 0.917$$

where *lna i*s regressed on its value lagged one year. The long-run estimate of the coefficient is −0.328. This gives an estimate of labour's share of 0.720.[17]

Consequently, when the lagged values of *lnV, lnw* and *lnL* are included in the above regression instead of the lagged value of *lna*, it is a foregone conclusion that they must be statistically significant. As the intercept in the regressions is a function of the factor share(s), it is not surprising that, in some cases, the goodness of fit is improved by the inclusion of the lagged variables. But it should be emphasized that this does not alter the interpretation that the estimated coefficients are simply reflecting the parameters of the identity.

We next estimated equation (2), namely $lnL = c_3 + b_4 \, lnV + b_5 \, lnw$, which could be viewed as an alternative specification of the labour demand function where the expected values of the coefficients b_4 and b_5 are 1 and −1 and the results are reported as equation (v) in Table 1. Given that factor shares are approximately constant, this equation is again an identity, and so the coefficients *must* take these values. From a neoclassical point of view, equation (2) has the advantage that it avoids the possible misspecification inherent in proxying technical change by a linear (or even non-linear) time trend. It has the disadvantage that the coefficient of *lnw* cannot now be interpreted as the elasticity of employment with respect to the real wage rate, but this can be calculated from the estimate of the intercept. It can be seen that the coefficient of *lnV* is 0.965 (with a t-statistic of 11.03) and *lnw* is −1.013 (−8.40) both of which are close to their expected values. (These are the long-run estimates when a one-period lag of *lnL* is included.) The estimate of the intercept is, however, poorly determined and is statistically insignificant.

The advantage of equation (3), $lnL = -(1-\sigma)lnA_0 + \sigma ln\delta + lnV - \sigma lnw - (1-\sigma)\lambda t$, is that it also avoids the need to calculate the rental price of capital and the capital stock, and can be derived from the more flexible CES production function. This is the functional form estimated by Lewis and MacDonald (2002) using quarterly Australian data. (As noted above, it is a marginal product revenue curve and not, strictly speaking, a labour demand curve. The elasticity can, however, be calculated from the estimated parameters.) In this case, as *L* is definitionally related to *V* and *w*, we can see that the estimates will still reflect those of the identity, albeit biased by the omission of *lnr* and *lnJ* as they are not adequately proxied by the time-trend. The results are reported in Table 1 equation (vi), and it can be seen that the omission of *lnr* and *lnJ* causes the goodness of fit to fall and the estimate of *lnw* is biased upwards, taking a value of −0.696 instead of −1.

Finally, we turn to the labour demand function when capital is kept constant, equation (4), The statistical fit is not particularly good, with the coefficient of *lnw* taking the wrong sign:

$$lnL = 8.632 \quad + 0.723 \ln J \quad + 0.447 lnw - 0.032t$$

$$(2.27) \quad\quad (2.97) \quad\quad (1.61) \quad\quad (-3.80)$$

$$\overline{R}^2 = 0.759, SER = 0.032, DW = 1.961$$

(The estimation method is the Exact AR(2) Newton-Raphson iterative method.). The reason is relatively straightforward. *lnJ* shows very little variation while the two omitted variables, *lnr* and *lnV* show considerable variability and this causes the poor statistical fit and the substantial degree of bias on the coefficients, especially of *lnw*.

We also have the problem that, at first glance, this specification seems to be incompatible with the identity. However, it will be recalled from equation (8) that *lnB + alnw + (1−a)lnr = lnA(t) = lnA$_o$ + λ(t)t*. If we use this equation to substitute for *lnA$_0$ + λt* in equation (13) (i.e. assuming that *λ = λ(t)*) and also use the relationship *lna = lnw +lnL −lnV,* we derive the identity given by equation (7). (It will be recalled that *a = α*.)

The wage elasticity and error-correction models

In this section we shall, for simplicity, confine ourselves to the marginal revenue product curve defined by equation (3) and used by Lewis and MacDonald (2002).

If we were dealing with a behavioural equation, then the question of whether the estimation of this specification gives rise to a spurious regression would be relevant. Consequently, assuming for the moment that it is a behavioural equation, we followed the procedure of Lewis and MacDonald and estimated equation (3) within an Autoregressive Distributed Lag model following the approach of Peseran and Shin (1999) and Pesaran, Shin and Smith (2001). The advantage of this approach is that it can be applied to models that contain a mixture of I(0) and I(1) variables and hence avoids the pre-testing problems involved with the standard cointegration analysis.

The error correction version of equation (3) is:

$$\Delta ln\ L = c_4 + \chi_0 t + \chi_1 \Delta ln\ L_{t−1} + \chi_2 \Delta ln\ V_{t−1} + \chi_3 \Delta ln\ w_{t−1} + \varsigma_1\ ln\ L_{t−1}$$
$$+ \varsigma_2\ ln\ V_{t−1} + \varsigma_3\ ln\ w_{t−1} + u_t \qquad (16)$$

The first test is the null hypothesis H$_0$: $\varsigma_1 = \varsigma_2 = \varsigma_3 = 0$. Using the critical bounds test devised by Peseran at al. (2001), the obtained F-value of 18.17 exceeds the upper bound the non-standard F-value of 5.76 and so the null hypothesis of there being no long-run relationship between *lnL*, *lnw* and *lnV* is rejected. Estimating equation (3) by an Auto Regressive Distributed Lag (ARDL) approach gives the long-run relationship as:

$$ln L = 8.948 − 0.009t + 1.636 lnV − 1.380 lnw$$
$$(2.87)\ (−2.22)\quad (3.96)\qquad (−2.22)$$

and the error-correction term from the specification including the lags – not reported here – is significant and is −0.280 with a t-value of −2.83. (As we are concerned with the long-run relationship, we do not report the specification with the lags.) However, the t-values of the long-run estimates are rather low and the estimated of the coefficient of *lnw* is rather small (−1.380), as opposed to the expected value of –1.000 implicit in the identity. The coefficients are also somewhat different from those reported in Table 1, equation (vi). This is due to the different estimation method.[18] However, the key point remains; *lnL*, *lnV* and *lnw* are definitionally related and even though the estimated values may diverge, to some extent, from those of the identity, they are still statistical artefacts. The error correction term is not the result of disequilibrium in the economic sense, but simply because the introduction of lagged values improves the goodness of fit of the (misspecified) identity.

The critique by Anyadike-Danes and Godley

A similar critique to the one discussed above (although with one or two key differences) has been put forward in an important, but somewhat neglected, paper by Anyadike-Danes and Godley (ADG) (1989a). Using a mark-up pricing model and an employment equation where real wages are not an argument, they show by simulation analysis that if the real wage is (erroneously) included in the regressions of the employment demand functions, its coefficient will still be negative and highly significant.

This paper has been surprisingly ignored in the literature and it is useful to compare its findings with the criticisms advanced above.[19] ADG specify four alternative models, but we shall only consider one of them here, namely their *Model 2*.

In this model, the authors postulate that there is a 'true' underlying employment function of the form:

$$\Delta ln \, L = \varphi \, [(C_1 + lnQ - lnS) - ln \, L_{-1}] \tag{17}$$

The variable *lnS* is the logarithm of the trend (and not the actual) rate of growth of productivity and there is a first-order partial adjustment process denoted by φ, which, in constructing the simulation data, is taken to be 0.3. *Q* denotes output in physical terms and C denotes a constant. In the long run, the growth of labour is determined by the growth of output and the exogenous trend growth of productivity. Prices are determined by current and lagged normal unit costs and the pricing equation is given by:

$$ln \, p = C_2 + \mu (lnw^n - lnS) + (1 - \mu)(lnw^n_{-1} - lnS_{-1}) \tag{18}$$

where μ describes the speed of adjustment. (Empirically, μ takes a value of about 0.75.) Combining equations (17) and (18), we obtain the hybrid function:

$$ln \, L = C_3 - \varphi \, (lnw^n - lnp) - \varphi \left(\frac{1-\mu}{\mu} \right)(lnw^n_{-1} - lnp_{-1}) + \varphi \, lnQ$$

$$+ \varphi \left(\frac{\mu}{1-\mu} \right)(p - p_{-1}) + \left(\frac{\mu}{(1-\mu)} \right)lnS_{-1} + (1-\varphi)ln \, L_{-1} \tag{19}$$

If the mark-up is just on current labour costs (i.e. $\mu = 1$), equation (19) reduces to:

$$ln \, L = C_1 - \varphi \, lnw + \varphi \, lnQ + (1-\varphi) \, ln \, L_{-1} \tag{20}$$

where $w = w^n - p$, so the long-run coefficients of *lnw* and *lnQ* are again −1 and 1.

If we were to test the neoclassical model by estimating equation (20), we would find the real wage term negative and highly significant, even though we know by construct that the real wage term has *no* role in determining the level of employment in a causal sense (it does not appear in the 'true' employment function).

ADG find that in estimating equation (20), the coefficients of all the independent variables are statistically significant and the long-run estimates of the coefficients of *lnQ* and *lnw* are 0.935 and −0.968 respectively.

ADG compare the performance of their model with the statistical estimations of employment, or labour demand, function by Bean, Layard and Nickell (1986), which takes the form:

$$\Delta ln \, L = c_5 + b_6 t + b_7 t^2 + b_8 (lnL_{-1} - lnJ) + b_9 lnw + b_{10}\Delta ln \, L_{-1} + b_{11}D \qquad (21)$$

where *D* represents real demand. This is constructed, as ADG point out, by first regressing the (logarithm of) output relative to capital stock on current and lagged fiscal and monetary policy variables (similarly scaled) together with a lagged dependent variable. ADG proxy *lnJ* by a log linear trend that rises by 1% per annum more than the trend of *lnQ* and a random fluctuation is then added to this trend. ADG's *Model 2* simply assumes that $D = lnV - LnJ$.

Bean et al.'s results for three countries, together with ADG's *Model 2* as a comparison, are reported in Table 2. The constant and time trend are not reported. It can be seen that the data are not able to discriminate between the two markedly different competing hypotheses, one that theoretically accords a causal role to real wages in determining the level of employment and the other that does not.

Our approach reinforces this conclusion. It will be recalled that the identity is given by equation (7), which subtracting $ln \, L_{-1}$ on both sides and rearranging it yields:

$$\Delta ln \, L \equiv -\frac{1}{a} ln \, B - (ln \, L_{-1} - ln \, J) - ln \, w + 0\Delta ln \, L_{-1} + \frac{1}{a}(lnV - ln \, J) + \frac{(1-a)}{a} lnr \quad (22)$$

For US manufacturing over the period 1960–1993, it will be recalled that labour's share, *a*, takes an average value of 0.73. Hence, the identity can be written as:

$$\Delta ln \, L \equiv C_4 - (ln \, L_{-1} - ln \, J) - ln \, w + 0\Delta ln \, L_{-1} + 1.37(lnV - ln \, J) + 0.37 lnr \quad (23)$$

where $lnV - lnJ = D$. The variable ΔlnL_{-1} has an expected coefficient of zero, as it should not be included in the identity. The results of estimating equation (23) are reported in Table 2, equation (i), where it can be seen that the shares are sufficiently constant to give a good statistical fit. As expected, the coefficient of ΔlnL_{-1} is statistically insignificant.

The identity is, of course, compatible with *any* underlying technology including one where the real wage has no role in determining the level of employment. Indeed, given the wide variety of industries, probably there is not a well-defined relationship between aggregate employment and aggregate output, even if both could be aggregated. However, from the identity, we see once again why there is likely to be a statistically significant relationship. In the Bean et al. formulation, the growth of the rate of profit does not appear, so this was dropped from the identity and, following Bean et al., a time trend was included instead. (It was found that a linear time trend gave the best fit and so, unlike in Bean et al., a quadratic time trend was not included.) The results are reported in Table 2 as equation (ii). It is interesting to note that the coefficient of ΔlnL_{-1} is now positive and statistically significant. It is positive in all three estimations by Bean et al. (but not in ADG's *Model 2*, where it was statistically insignificant.)[20]

As we have noted, the variable *(lnV-lnJ)* may be interpreted as a proxy for Bean et al.'s measure of demand, and the coefficient of this variable falls to 0.57 in Table 2, equation (ii), which is not far off Bean et al's estimates of the coefficient of *D*, especially for the UK and Germany. It should be emphasized that the differences in the values of the coefficients from those of the full identity are just due to omitted variable bias and the inclusion of irrelevant variables. To this extent they may be

Table 2.   Estimates of the Bean et al. (1986), Anyadike-Danes and Godley (1989a) and related models. Dependent variable $\Delta ln\,L$.

	Bean et al.'s estimates				Authors' estimates			Expected values from the identity
	UK	France	Germany	ADG's Model 2	Equation (i)	Equation (ii)	Equation (iii)	
$lnL_{-1}-lnJ$	−0.63 (−4.8)	−0.28 (−3.5)	−0.64 (−2.1)	−0.32 (−23.2)	−1.03 (−64.4)	−0.86 (−9.0)	−0.66 (−14.1)	−1.00
lnw	−0.40 (−2.0)	−0.17 (−3.4)	−0.53 (−3.5)	−0.23 (−10.0)	−1.05 (−49.7)	−0.45 (−3.5)	−0.12 (−1.9)	−1.00
ΔlnL_{-1}	0.45 (2.3)	0.22 (1.0)	0.51 (1.8)	−0.01 (−0.4)	−0.01 (−0.9)	0.16 (1.9)	−0.07 (−1.4)	0.00
D	0.50 (7.1)	0.14 (1.3)	0.46 (1.4)	–	–	–	–	–
$lnV-lnJ$ (= D)	–	–	–	0.31 (26.8)	1.39 (68.8)	0.57 (10.6)	–	1.37
$lnCAP$	–	–	–	–	–	–	0.85 (21.6)	–
lnr	–	–	–	–	−0.38 (−41.7)	–	–	−0.37
t	n.r.	n.r.	n.r.	n.r.	–	−0.01 (−4.8)	−0.02 (−13.0)	–

Notes: n.r. denotes not reported. Bean et al. (1986) use a linear and quadratic time trend. The 'expected values' in the last column are those of the full identity given by equation (23). Figures in parentheses are t-values.
Sources: Bean et al. (1986), Anyadike-Danes and Godley (1989a, 183) and authors' estimates.

regarded as purely coincidental – there is certainly nothing in production theory that suggests that the biases ought to be of either the sign or the order of magnitude that they take.

Finally, we replaced *(lnV-lnJ)* by *lnCAP*, which is the logarithm of a capacity utilization variable.[21] It can be seen that it is a good, but not perfect proxy, for *lnV-lnJ* and as a result the estimates of the coefficients change somewhat. But the negative coefficient of *lnw* is still statistically significant although its absolute value is low.

ADG compare their simulation results with further empirical estimates from other studies of the side relations derived from the neoclassical production function and we have done likewise. The results are not reported here, but the statistically significant negative coefficient of real wages, not surprisingly, is also found in these other neoclassical studies.

Conclusions

This paper has shown that the test of the neoclassical hypothesis that employment and the wage rate are inversely related, i.e. the estimation of the labour demand function and the marginal revenue product curve, faces insoluble problems. This is due to the fact that empirical applications use value data as opposed to physical quantities. Since value data and employment are linked through an accounting identity, we show that estimation of the labour demand function and the marginal product revenue curve with these data will always yield a negative relationship between the level of employment and the real wage.

In fact, the data must normally give a good statistical fit to either the neoclassical labour or capital demand functions even when, because of the multitude of firms with very different production functions, there might not be any well-defined aggregate production function or factor demand functions at all. Moreover, the negative coefficient on the wage term in the labour demand equations (and the marginal revenue product curve) is determined solely by the underlying identity. However, we have also shown with our data that, even when the factor shares are roughly constant, some of the specifications do not give near perfect statistical fits. This may give rise to the mistaken belief that a behavioural equation is being estimated.

We have taken the simplest labour and capital demand functions because these most clearly demonstrate the problems involved. But the problems posed carry through to more complicated factor demand functions. This has been shown by reconsidering the argument of Anyadike-Danes and Godley (1989a) and we have confirmed the importance of their arguments, which are similar, although not identical, to the ones outlined in this paper.

It is clear that no reliance can be placed on estimates of the wage elasticities in formulating economic policy. Arguments that an increase in the real wage rate will necessarily lead to a fall in the level of employment cannot be inferred from the statistical estimates of the elasticity of employment with respect to the real wage. To base policy solely on this evidence may have unforeseen and unwanted consequences.

Notes

1. The quotation is from T.S. Eliot's poem, 'Little Gidding'.
2. Some Keynesians, while accepting the marginal productivity theory of factor pricing, would dispute this line of reasoning. They argue that while there is an inverse relationship between the wage rate and the level of employment (because of diminishing returns), the causation is not that of the neoclassicals. It is the level of demand that determines the demand for labour, which in turn determines the real wage (see, for example, Davidson 1983; Thirlwall 1993). We shall not pursue this argument here.
3. This is because under the usual neoclassical assumptions, the rate of technical progress is given by the growth of the real factor prices weighted by their factor shares. This means the specifications become exact identities.
4. However, an increase in the money wage may increase the price of output relative to that of other goods and services. Assuming a demand equation for output as $Q = cp^{-\pi}$, where $\pi \geq 0$ is the absolute value of the elasticity of product demand and c is a constant, the wage elasticity of the demand for labour becomes $-(1-a)\sigma - a\pi$. As precise estimates of the price elasticity of demand for output are difficult to obtain, the demand side is normally ignored in the literature, which is equivalent to assuming that the demand for the industry's output is either completely price inelastic or supply constrained.

   Another implicit assumption is that the elasticity of supply of capital goods and structures is infinite. If it is not, the expression for the wage elasticity becomes more complicated with the elasticity of supply of the capital stock being one of its arguments. Again, it is normally assumed that this is infinite, in which case the elasticity of demand for labour is again equal to $-(1-a)\sigma$ or alternatively to $-(1-a)\sigma - a\pi$.
5. The argument holds equally if we use the identity for gross output, or sales, instead.
6. If there are measurement errors in the calculation of ρ, this will affect the value of r_{nc}, which is calculated residually.
7. A serious problem is that there is no way of testing whether ρ (which is calculated using a number of restrictive assumptions and suffers from serious aggregation problems) correctly measures the competitive rate of profit. It can be compared with the *ex post* rate of profit but it is impossible to determine whether any difference is due to the state of competition or to errors inherent in calculating ρ.
8. In a perceptive comment, Jorgenson and Griliches (1967, 257 fn 2, emphasis in the original) note:

   > The answer to Mrs. Robinson's … rhetorical question, 'what units is capital measured in?' is dual to the measurement of the price of capital services. Given either an appropriate measure of the flow of capital services or a measure of its price, the other measure may be obtained from the value of income from capital. Since this procedure is valid only if the necessary conditions for producer equilibrium are satisfied, the resulting of quantity may not be employed to *test* the marginal theory of distribution, as Mrs. Robinson and others have pointed out.

   However, what they have overlooked is that this holds regardless of whether or not the conditions for producer equilibrium exist, as we show in the text.
9. It could also be argued that it is not clear that large oligopolistic firms necessarily base their investment and labour-hiring decisions on the rental price of capital, which is derived from an untested optimizing microeconomic model. The rate of profit of a firm, which closely correlates with its internal funds from which most investment is financed, may actually be of greater importance (as, indeed, is the state of expectations about future demand). Thus, the labour demand function is correctly specified using r, the *ex post* rate of profit. However, it must be emphasized that the argument we are making in this paper does not rely on this assumption. Moreover, equations (2), (3) and (4) do not use the rental price of capital.
10. Note that all we require is for factor shares to be constant for a Cobb-Douglas to give a perfect fit to the identity. This does not have to result from a constant mark-up. The Kaldorian theory of distribution, for example, will give the same result. The seminal article is Kaldor (1956).
11. Recall the discussion above about the difference between the rate of profit and rental price of capital if markets are not competitive.

12. In fact, any underlying physical production functions will give rise to an aggregate Cobb-Douglas production function if factor shares are constant.
13. Under the usual neoclassical assumptions the dual of total factor productivity growth, when factor shares are constant, is given by $\lambda(t) \equiv a\hat{w} + (1-a)\hat{r}$ and also $lnA(t) = lnB + alnw + (1-a)lnr$.
14. As we are illustrating a theoretical point, the exact period of the data set used is not particularly important.
15. As we are not dealing with behavioural equations, the exact specifications in terms of lags etc. in the empirical results are determined by the goodness of fit.
16. We are grateful to a referee for posing this question.
17. Simply regressing *lna* on a constant gives a coefficient of -0.318 with a t-ratio of -57.21. This gives a value of labour's share of 0.728.
18. The long-term estimates of Lewis and MacDonald (2002) using Australian quarterly data for the whole economy over the period 1961–1998 are:

$$lnL = -0.0981 - 0.0031t + 1.058lnQ - 0.446lnw$$
$$(-0.26) \quad (-8.31) \quad (17.05) \quad (-6.20)$$

19. There was a rejoinder by Nickell (1989), but it is difficult not to agree with Anyadike-Danes and Godley (1989b) that, while it raised some interesting issues, it did not address their argument.
20. It is positive and statistically significant in their *Model 1*, which we have not discussed.
21. We are grateful to Anwar Shaikh for providing us with his capacity utilization estimates.

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