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On the interpretation of coefficients in multiplicative-logarithmic functions: a reconsideration

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This paper shows that previous arguments about the problems in interpreting the coefficients in multiplicative-logarithmic functions, derived from an arbitrary rebasing of the series, are incorrect. In the specific case of the translog production function, the tests for the CES, Cobb–Douglas and constant returns to scale restrictions are shown to be invariant to any rescaling.

I. INTRODUCTION

Hunt and Lynk (1993) showed that the size, sign and significance of the estimated coefficients in multiplicative-logarithmic functions such as the translog are not invariant to the units of measurement utilized. The authors concluded: '... It is reasonably clear that careful scrutiny is required when interpreting the results from multiplicative logarithmic functions' (Hunt and Lynk, 1993, p. 737). And: 'This paper has illustrated the difficulty of interpreting coefficients and conducting reliable significance tests in multiplicative-logarithmic functions of the type commonly found in applied econometric analysis' (Hunt and Lynk, 1993, p. 738). If correct, Hunt and Lynk's results would have an important impact on the literature on production functions, and would make us reconsider the findings of a large number of studies using translog production functions. However, this comment will prove that while Hunt and Lynk's algebra is correct, their arguments are flawed and there is no problem in using a translog. The authors showed that the multiplicative-logarithmic forms

$$\ln Y = \alpha_0 + \alpha_1 \ln X_1 + \alpha_2 \ln X_2 + \alpha_3 \ln X_1 \ln X_2 \qquad (1)$$

$$\ln Y = \beta_0 + \beta_1 \ln X_1^* + \beta_2 \ln X_2^* + \beta_3 \ln X_1^* \ln X_2^* \qquad (2)$$

where

$$X_1^* = X_1/a \qquad (3)$$

$$X_2^* = X_2/b \qquad (4)$$

with a and b being two arbitrary constants, yield parameters related by

$$\alpha_0 = \beta_0 + \beta_3 \ln a \ \ln b - \beta_1 \ln a - \beta_2 \ln b \qquad (5)$$

$$\alpha_1 = \beta_1 - \beta_3 \ln b \qquad (6)$$

$$\alpha_2 = \beta_2 - \beta_3 \ln a \qquad (7)$$

$$\alpha_3 = \beta_3 \qquad (8)$$

Since the estimated coefficients of X_1 and X_2 are different in both regressions (see Equations 6 and 7), Hunt and Lynk argued that the results of the estimations are not invariant to the units of measurement. Therefore, the econometric estimation of the translog production function is probably a useless and wrong exercise.

II. SOME ALGEBRA

While Hunt and Lynk's algebra is correct and it is true that these two parameters of the production function are affected by the units of measurement, this is an irrelevant issue for

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analysis purposes. First, notice that the only two parameters affected by the rescaling are the estimates of $\ln X_1$ and $\ln X_2$ (and the constant). The estimate of $(\ln X_1 \ln X_2)$ is invariant to the rescaling. In the case of the translog production function, one has to recall that the parameters do not have a direct interpretation, as opposed to the Cobb–Douglas, for example, where they are the elasticities. Thus, if one wants to use a translog to estimate the rate of disembodied technological progress, for example, the estimate of the time trend would not be affected by the rebasing, a, Table 1 in Hunt and Lynk shows. Likewise, the summary statistics of the regression, i.e. fit, Durbin–Watson, and standard error of the regression (SER), are the same, also shown in the said table. This implies that the residuals of both regressions must be the same. Finally, as the authors themselves point out, the labour and capital elasticities are the same. In fact, Regressions 1 and 2 are the same. This can be seen substituting Expressions 5–8 in Equation 1

$$\ln Y = \beta_0 + \beta_1[\ln X_1 - \ln a] + \beta_2[\ln X_2 - \ln b] + \beta_3[\ln X_1 - \ln a][\ln X_2 - \ln b] \tag{2'}$$

which is equivalent to Expression 2. No wonder why all statistics are identical regardless of the rebasing (nine different rescalings in Hunt and Lynk's paper). This implies that all regressions must lead to the same conclusions in any analysis. To see this more clearly, suppose we estimate the translog production function

$$\ln Y_t = c + \alpha_1 \ln L_t + \alpha_2 \ln K_t + \alpha_3(\ln L_t)^2 + \alpha_4(\ln K_t)^2 + \alpha_5 \ln L_t \ln K_t \tag{9}$$

and also assume α_3 happens to be statistically insignificant. Certainly this would not be a reason to eliminate it from the regression and rerun it without $(\ln L_t)^2$. Likewise, one does not usually test the single null hypothesis $H_0 : \alpha_1 = 0$. This is not an economically meaningful restriction. To see further the implications of this analysis, notice that the test for the CES restriction yields the same statistic, independently of the rebasing. The null hypothesis is $H_0 : \alpha_3 = \alpha_4 = -(\alpha_5/2)$. Imposing this restriction on Expression 9 yields

$$\ln Y = c + \alpha_1 \ln L + \alpha_2 \ln K - \frac{\alpha_5}{2}[\ln K - \ln L]^2 \tag{10}$$

and if we rebase Y, L and K, the corresponding CES is

$$\ln Y = d + \beta_1 \ln L^* + \beta_2 \ln K^* - \frac{\beta_5}{2}[\ln K^* - \ln L^*]^2 \tag{10'}$$

where the asterisk denotes the rebased series (the rebasing of the dependent variable only affects the constant term). The OLS estimates in the above two regressions 10 and 10′ will yield $c \neq d$, $\alpha_1 \neq \beta_1$, $\alpha_2 \neq \beta_2$, $-(\alpha_5/2) = -(\beta_5/2)$. But the regressions are equivalent, since they yield the same statistics. This can be seen by substituting $L^* = L/a$ and $K^* = K/b$ into Equation 10′ (the rebasing of the dependent variable only affects the constant term). This yields

$$\ln Y = A + k_1 \ln L + k_2 \ln K - \frac{\beta_5}{2}\Big[\ln L - \ln K\Big]^2 \tag{11}$$

where

$$A = d - \beta_1 \ln a - \beta_2 \ln b - \frac{\beta_5}{2}(\ln a)^2 - \frac{\beta_5}{2}(\ln b)^2 + \beta_5(\ln a)(\ln b) \tag{12a}$$

$$k_1 = \beta_1 + \beta_5 \ln a - \beta_5 \ln b \tag{12b}$$

$$k_2 = \beta_2 + \beta_5 \ln b - \beta_5 \ln a \tag{12c}$$

Comparing Equation 10 with 11 we have $c = A$, $\alpha_1 = k_1$, $\alpha_2 - k_2$, $-(\alpha_5/2) = -(\beta_5/2)$; and substituting Expressions 12a–12c into Equation 10 we obtain Equation 10′.

The Cobb–Douglas case arises from the translog after imposing the restriction $\alpha_3 = \alpha_4 = \alpha_5 = 0$. Algebraically,

$$\ln Y = c + \alpha_1 \ln L + \alpha_2 \ln K \tag{13}$$

and with the rebasing

$$\ln Y = d + \beta_1 \ln L^* + \beta_2 \ln K^* \tag{13'}$$

Once again, Regressions 13 and 13′ are equivalent. In this case, $c \neq d$, $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_2$, and all statistics are equivalent.

A third example that an arbitrary rescaling of the variables in the translog production function does not affect the interpretation of the results is the test for constant returns to scale. Since the output elasticities are invariant to the rescaling, it should be obvious that a test for the returns (sum of the elasticities) to scale is unaffected. Hunt and Lynk (1993) showed in their example that the elasticities are invariant. The algebra is as follows:

$$\frac{\partial \ln Y}{\partial \ln L} = \alpha_1 + 2\alpha_3 \ln L + \alpha_5 \ln K \tag{14}$$

and after rescaling

$$\frac{\partial \ln Y}{\partial \ln L^*} = \beta_1 + 2\beta_3 \ln L^* + \beta_5 \ln K^* \tag{14'}$$

Substituting $L^* = L/a$ and $K^* = K/b$ into Equation 14 yields

$$\frac{\partial \ln Y}{\partial \ln L^*} = B + 2\alpha_3 \ln L + \alpha_5 \ln K \tag{15}$$

where

$$B = \alpha_1 - 2\alpha_3 \ln a - \alpha_5 \ln b \tag{16}$$

If we now compare Equations 14 and 15 we see that $\alpha_1 = B$, and if we substitute Equation 16 into Equation 14 we obtain Equation 14'. We can now complete the argument

Table 1. *OLS estimates of a translog production function for Singapore 1970–90, and tests of CES, Cobb–Douglas (C-D), and constant returns to scale (CRTS) restrictions*

	α_0	α_1	α_2	α_3	α_4	α_5
I	−52.26	35.80	−15.44	−4.75	−0.58	3.64
	(−0.98)	(1.56)	(−2.27)	(−1.90)	(−2.45)	(3.64)
II	−0.029	1.18	0.32	−4.75	−0.58	3.64
	(4.53)	(5.25)	(4.54)	(−1.90)	(−2.45)	(2.40)
	R^2	DW	SER			
I, II	0.999	1.03	0.0039519			
CES	c	α_1	α_2	$-(\alpha_5/2)$		
I	0.95	1.78	−0.56	0.18		
	(1.47)	(7.93)	(−3.41)	(6.32)		
II	−0.024	0.73	0.48	0.18		
	(−2.41)	(5.06)	(10.22)	(6.32)		
	R^2	DW	SER	$H_0: \alpha_3 = \alpha_4 = -(\alpha_5/2)$		
I, II	0.996	0.68	0.012561	χ^2_2 = 32.67 (Wald)		
C-D	c	α_1	α_2			
I	0.33	0.69	0.43			
	(0.29)	(2.69)	(5.19)			
II	0.008	0.69	0.43			
	(0.53)	(2.69)	(5.19)			
	R^2	DW	SER	$H_0: \alpha_3 = \alpha_4 = \alpha_5 = 0$		
I, II	0.989	0.24	0.042124	χ^2_3 = 19.02 (Lagrange)		
ELASTIC	α_1	$2\alpha_3$	α_5			
I	1.1839	−9.494	3.6467			
II	35.8049	−9.494	3.6467			
CRTS	c	α_1	α_5			
I	2.32	1.41	−0.17			
	(10.85)	(8.58)	(−5.50)			
II	−0.0099	0.42	−0.17			
	(−1.18)	(19.20)	(−5.50)			
	R^2	DW	SER	$H_0: \alpha_1 + \alpha_2 = 1; 2\alpha_3 + \alpha_5 = 2\alpha_4 + \alpha_5 = 0$		
I, II	0.98	0.71	0.016145	χ^2_3 = 46.28 (Wald)		

I Original series of Y, K and L
II Normalized series by dividing by their sample means

by showing the invariance of the test for constant returns to scale. The null hypothesis of this test is $H_0: \alpha_1 + \alpha_2 = 1$; $2\alpha_3 + \alpha_5 = 2\alpha_4 + \alpha_5 = 0$. Imposing the restriction in Equation 9 yields

$$\ln\frac{Y}{K} = c + \alpha_1 \ln\frac{L}{K} + \frac{\alpha_5}{2}\left[2(\ln L)(\ln K) - (\ln L)^2 - (\ln K)^2\right] \tag{17}$$

and with the rebasing

$$\ln\frac{Y}{K^*} = d + \beta_1 \ln\frac{L^*}{K^*} + \frac{\beta_5}{2}\Big[2(\ln L^*)(\ln K^*) - (\ln L^*)^2 - (\ln K^*)^2\Big] \tag{17$'$}$$

In this case, plugging $L^* = L/a$ and $K^* = K/b$ into Equation 14′ and rearranging terms yields

$$\ln\frac{Y}{K} = B + k_3 \ln\frac{L}{K} + \beta_5\Big[2(\ln L)(\ln K) - (\ln L)_2 - (\ln K)^2\Big] \tag{18}$$

where

$$B = d - \beta_1 \ln a - (1-\beta_1)\ln b - \frac{\beta_5}{2}(\ln a)^2 - \frac{\beta_5}{2}(\ln b)^2 + \beta_5(\ln a)(\ln b) \quad (19a)$$

$$k_3 = \beta_1 + \beta_5 \ln a - \beta_5 \ln b \quad (19b)$$

Comparing Equations 17 and 18 we have $c = B$, $\alpha_1 = k_3$, $\alpha_5/2 = \beta_5/2$; and substituting Expressions 19a and 19b into Equation 17 we obtain Equation 17′.

III. AN ILLUSTRATION: THE TRANSLOG PRODUCTION FUNCTION

To see this with an example, we estimate the translog production function Equation 9 with data provided by Chen (1991) for Singapore. The results are shown in Table 1. Equation I uses the original series, while Equation II uses normalized data dividing each series by its mean. The results indicate that the rescaling affects the estimates of α_0, α_1, and α_2, but not the other three parameters. The regression statistics are not affected either. The second part of the Table shows the estimate of the CES production function, leading to the same conclusions: although three of the parameter estimates are different, the regressions are the same; and estimating the translog, the test for the CES ($H_0: \alpha_3 = \alpha_4 = -(\alpha_5/2)$) yields the same result in both equations. The same can be said about the Cobb–Douglas. The two regressions only differ in the constant term and when the restriction for Cobb–Douglas is imposed in the translog ($H_0: \alpha_3 = \alpha_4 = \alpha_5 = 0$) the result is the same statistic. Finally, the same occurs when one tests for constant returns to scale in the translog production function ($H_0: \alpha_1 + \alpha_2 = 1, 2\alpha_3 + \alpha_5 = 2\alpha_4 + \alpha_5 = 0$).

IV. SUMMARY

This paper has shown that the results and arguments in Hunt and Lynk (1993) are erroneous. Although some of the estimated coefficients of translog production functions vary with an arbitrary rebasing, this does not affect the interpretation of the results, and tests for the Cobb–Douglas, CES and constant returns to scale restrictions are not influenced.

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