"A THEORY OF PRODUCTION"[1]

THE ESTIMATION OF THE COBB-DOUGLAS FUNCTION: A RETROSPECTIVE VIEW

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As Solow once remarked to me, we would not now be concerned with the question [the existence of the aggregate production function] had Paul Douglas found labor's share of American output to be twenty-five per cent and capital's share seventy-five instead of the other way around [Fisher, 1969, 572].

I hope that someone skilled in econometrics and labor will audit and evaluate my critical findings [Samuelson, 1979, 934].

INTRODUCTION

Despite honoring Douglas's important contributions to economics, to the point of arguing that "If Nobel Prizes had been awarded in economics [...], Paul H. Douglas would probably have received one before World War II for his pioneering econometric attempts to measure marginal productivities and quantify the demands for factor inputs" [Samuelson, 1979, 923], Samuelson [1979] offered a grave assessment of the empirical significance of the Cobb-Douglas production function and the associated marginal productivities. The argument that Samuelson sketched is that the parameters of what is believed to be an aggregate production function may be no more than the outcome of an income distribution identity. It is ironic that this same argument had been put forward very clearly by other scholars well before Samuelson. The profession, however, ignored it. The argument had appeared in Phelps Brown [1957]; Simon and Levy [1963] and Shaikh [1974]. Moreover, Simon [1979] thought that the argument was so important that he discussed it in his Nobel Lecture. Shaikh [1980] provides one of the most comprehensive treatments of the early discussions of the argument. More recent discussions and extensions are provided by Felipe and McCombie. See references.

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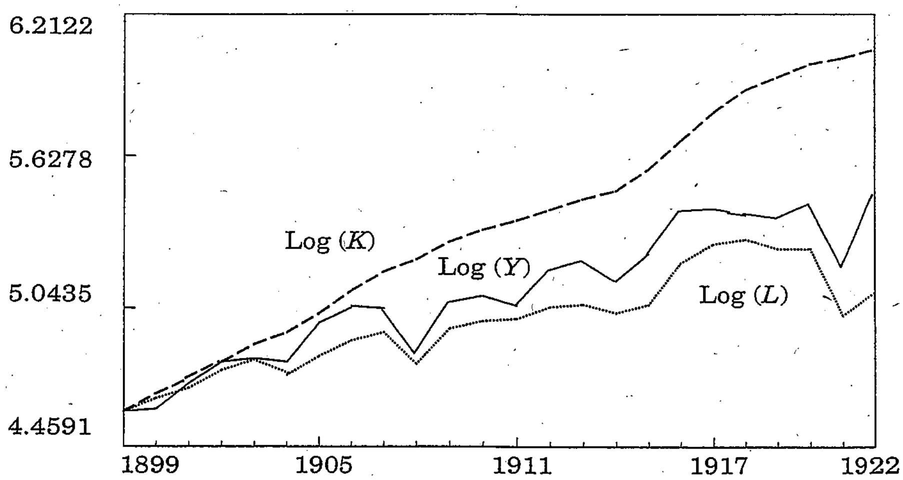
The Cobb-Douglas production function is still today the most ubiquitous form in theoretical and empirical analyses of growth and productivity. The estimation of the parameters of aggregate production functions is central to much of today's work on growth, technological change, productivity, and labor. Empirical estimates of aggregate production functions are a tool of analysis essential in macroeconomics, and important theoretical constructs, such as potential output, technical change, or the demand for labor, are based on them.

This paper takes up Paul Samuelson's invitation (quoted above) to evaluate empirically his arguments; and it does so by using the original data set of Cobb and Douglas [1928].

The origins of the Cobb-Douglas form date back to the seminal work of Cobb and Douglas [1928], who used data for the U.S. manufacturing sector for 1899-1922 (although, as Brown [1966, 31], Sandelin [1976], and Samuelson [1979] indicate, Wicksell should have taken the credit for its "discovery", for he had been working with this form in the 19th century).

At the time, Douglas was studying the elasticities of supply of labor and capital, and how their variations affected the distribution of income [Douglas, 1934]. To make sense of and interpret the numbers obtained, Douglas needed a theory of production. He began by plotting the series of output (Day index of physical production), labor (workers employed), and fixed capital on a log scale. He noted that the output curve lay between the two curves for the factors, and tended to be approximately one quarter of the distance between the curves of the two factors (Figure 1).

FIGURE 1
Cobb-Douglas [1928] Data Set (Logarithmic Scale)



With the help of Cobb, Douglas estimated econometrically what is known today as the "Cobb-Douglas" production function. This seminal paper plays a paramount role in the history of economics, since it was the first time that an aggregate production function was estimated econometrically and the results presented to the economics profession, although as Levinsohn and Petrin [2000] note, economists had been relating output to inputs since the early 1800s. The estimated OLS regression $Q_t = B(L_t)^{\alpha}(K_t)^{\beta}$, where Q_t, L_t, and K_t represent (aggregate) output, labor, and capital, respectively, and B is a constant, showed that the elasticities came remarkably close to the observed factor shares in the American economy, that is, $\alpha = 0.75$ for labor and $\beta = 0.25$ for capital (Cobb and Douglas estimated the regression imposing constant returns to scale in per capita terms. Standard errors and R were not reported). These results were taken, implicitly, as empirical support for the existence of the aggregate production function, as well as for the validity of the marginal productivity theory of distribution.

Douglas [1967] documents that the Cobb-Douglas production function was received with great hostility. The attacks were from both the conceptual and econometric points of view. At the time, many economists criticized any statistical work as futile (it was argued that the neoclassical theory was not quantifiable). Others launched an econometric critique against this work, noticing problems of multicollinearity, the presence of outliers, the absence of technical progress, and the aggregation of physical capital. These issues were raised and discussed by Samuelson [1979].

In this paper we fully develop the argument that all the estimation of the Cobb-Douglas function does is to reproduce the income accounting identity that distributes value added between wages and profits. If this is the case, one must seriously question not only Cobb and Douglas' original results, but the plethora of estimations carried out during the last seven decades.

To begin, one must remember that two strands of the literature questioned long ago the notion of an aggregate production function from a theoretical point of view. These are summarized and discussed by Felipe and Fisher [2003]. One strand is the so-called Cambridge (UK) – Cambridge (USA) capital debates. In a seminal paper, Joan Robinson [1953-54] asked the question that triggered such debate: "In what unit is 'capital' to be measured?" Robinson was referring to the use of "capital" as a factor of production in aggregate production functions. Because capital goods are a series of heterogeneous commodities (investment goods), each having specific technical characteristics, it is *impossible* to express the stock of capital goods as a homogeneous physical entity. Robinson claimed that only their values can be aggregated. Therefore, it is impossible to get any notion of capital as a measurable quantity independent of distribution and prices.[2]

The second strand of the literature that questions the notion of aggregate production function is known as the aggregation literature. This one studies the conditions under which neoclassical micro production functions can be aggregated into a neoclassical aggregate production function. The best exponent of this work is Franklin Fisher, whose extensive work began in the mid 1960s and was compiled in Fisher [1993]. Fisher concluded that the conditions for successful aggregation of micro production functions into an aggregate production function with neoclassical properties are so stringent that one should not expect any real economy to satisfy them. The conclusions of the Cambridge debates and the aggregation literature are so damaging for the notion of an aggregate production function that one wonders why it continues being used. The answer of the defenders of the use of aggregate production functions, as Cohen and Harcourt [2003, 209] note, is that "these 'lowbrow' models remain heuristically important for the intuition they provide, as well as the basis for empirical work, that can be tractable, fruitful and policy-relevant." If Samuelson [1979] was correct, however, this instrumentalist position is problematic and indefensible.

The rest of the paper is structured as follows. In the next section we re-estimate the Cobb-Douglas function with the original Cobb-Douglas [1928] data set, taken from Pesaran and Pesaran [1997, data file CD.FIT] and reproduced in Table 1.

TABLE 1
Output, Labor, and Capital

Year	Output	Labor	Capital
1899	100	100	100
1900	101	105	107
1901	112	110	114
1902	122	118	122
1903	124	123	131
1904	122	116	138
1905	143	125	149
1906	152	133	163
1907	151	138	176
1908	126	121	185
1909	155	140	198
1910	159	144	208
1911	153	145	216
1912	177	152	226
1913	184	154	236
1914	169	149	244
1915	189	154	266
1916	225	182	298
1917	227	196	335
1918	223	200	366
1919	218	193	387
1920	231	193	407
1921	179	147	417
1922	240	161	431

Source: Pesaran and Pesaran [1997; data file CD.FIT].

We point out a series of problems, in particular the poor results obtained once an exponential time trend is introduced in the regression in order to capture the evolution of technical progress. Most likely, if Cobb and Douglas had introduced the trend in their function, their results would not have been published, and, as Solow pointed out, we would not now be discussing aggregate production functions. We then provide a simple interpretation of what the estimated parameters of the aggregate Cobb-Douglas production function are. As Samuelson [1979] conjectured, this explanation is that all the aggregate Cobb-Douglas function regression captures is the path of the value added accounting identity according to which value added equals the sum of the wage bill plus total profits. In this section, the Cobb-Douglas form is simply derived as an algebraic transformation of the identity. This transformation embodies the result that the estimated parameters *must be* the factor shares. Then we take a second look at the Cobb-Douglas [1928] data set in light of the discussion in the previous section and solve the conundrum regarding the time trend. We continue by asking whether the aggregate production function provides an adequate framework to test for constant returns to scale and competitive markets through the marginal productivities. This is an important question because Douglas was convinced that the coincidence of the estimated coefficients with the actual factor shares received by labor and capital corroborated the neoclassical theory of income distribution. This issue is relevant for today's work.

A FIRST LOOK AT THE EMPIRICAL EVIDENCE

Table 2 reports several regressions and results very similar to those obtained by Cobb and Douglas. These results will help us highlight some of the initial criticisms their work faced. The first regression reports unrestricted estimates of the regression $Q_t = B(L_t)^{\alpha}(K_t)^{\beta}$ in logarithms. The results indicate that the constant returns to scale restriction is not rejected by the data. These results are sufficiently good to validate Cobb and Douglas's point. In particular, the two elasticities are relatively close to the observed factor shares in output, and thus add up to one, indicating constant returns to scale (chi-square test). The second regression shows the estimates of the regression in per capita terms and imposing the constant returns to scale restriction, as Cobb and Douglas estimated it initially. The implicit elasticity of labor is 0.751 with a t-value of 16.15.

TABLE 2
Cobb-Douglas Regression, I
(1899-1922 unless otherwise indicated; OLS estimates)

1. $\ln Q_t = c + \alpha \ln L_t + \beta \ln K_t$		Constant	α	β
		–0.18	0.807	0.233
		(–0.41)	(5.56)	(3.67)
$R^2 = 0.975$; D.W. = 1.52; $\chi^2_1 = 0.19$				
2. IN PER CAPITA TERMS: $\ln(Q_t/L_t) = (\alpha + \beta - 1)\ln L_t + \beta \ln(K_t/L_t)$		Constant	$1 - \alpha - \beta$	β
		–0.18	0.04	0.233
		(–0.41)	(0.44)	(3.67)
$R^2 = 0.636$; D.W. = 1.52				
3. $\ln Q_t = c + \lambda T + \alpha \ln L_t + \beta \ln K_t$	Constant	λ	α	β
	2.81	0.0468	0.906	–0.526
	(2.03)	(2.26)	(4.48)	(–1.54)
$R^2 = 0.966$; D.W. = 1.63				
4. $q_t = \lambda + \alpha \ell_t + \beta k_t$		λ	α	β
		0.10	1.39	–1.51
		(2.77)	(8.53)	(–2.53)
$R^2 = 0.80$; D.W. = 1.67; $\chi^2_1 = 4.59$				
5. ESTIMATION PERIOD: 1899-1920: $\ln Q_t = c + \alpha \ln L_t + \beta \ln K_t$		Constant	α	β
		–0.79	1.09	0.08
		(–1.42)	(4.88)	(0.73)
$R^2 = 0.972$; D.W. = 1.21; $\chi^2_1 = 2.13$				

Chi-square test (χ^2_1): H_0: $\alpha + \beta = 1$ (critical value 5 percent significance level: 3.84). t-statistics in parentheses.

These estimates, however, soon ran into the criticism that Cobb and Douglas had not included a measure of technical progress in their equation. Samuelson [1979, 924] claims that Schumpeter was shocked that the Cobb-Douglas formula did not allow for technical progress.[3] The solution proposed was to add an exponential time trend to the regression. We therefore re-estimated it, including an exponential time trend (T), that is, $Q_t = Be^{\lambda T}(L_t)^{\alpha}(K_t)^{\beta}$, in logarithms and unrestricted. The results, shown in the third regression of Table 2, are somewhat surprising in that now the coefficient of the index of capital is negative and insignificant. Nevertheless, despite these results, the regression displays a fit of 0.966. The negative sign of the capital coefficient remains in the fourth regression, when the equation is estimated in growth rates (and worse, the coefficient now is statistically significant). Although the fit is lower, it is still a not negligible 0.80. Finally, to test for stability, the fifth regression was estimated for 1899-1920. One might argue that the years 1921-22 could be taken to be outliers since output dropped by almost a quarter and then recovered. Although the results are very poor (see elasticities), the fit continues to be very high. And the recursive and rolling estimations of this regression (not shown but available upon request) prove its fragility. Only the regression with the complete period yields sensible results. We thus conclude that if computer technology had allowed Cobb and Douglas to perform the analysis carried out here, their results would have been dismissed.

We must note that the result of a negative capital coefficient is not news to those who have estimated Cobb-Douglas production functions. In fact, it is a standard finding [Lucas, 1970; Romer, 1987; Klette and Griliches, 1996; Griliches and Mairesse, 1998]. How can these results be interpreted if one insists that a production function has been estimated? Why does the regression work better without a time trend, which proxies the evolution of *technical progress*? Do we have to open the econometrics and data-mining toolkits and "torture" the data until more acceptable results appear (for example, endogeneity of the regressors, unit roots and possible cointegration issues, lack of adjustment of the stock of capital for utilization capacity)? Or, do we need to develop a new growth model to justify a negative (or zero) elasticity for capital? We believe a more parsimonious explanation can be provided.

THE INCOME ACCOUNTING IDENTITY AND THE AGGREGATE PRODUCTION FUNCTION

As indicated in the Introduction, the argument of this paper is that all estimations of aggregate production functions do is to reproduce the distribution income accounting identity. In this section we develop the argument. To begin, let us write the income accounting identity for real value added (Q), that is, the difference between gross output and intermediate materials, at time t, which equals the sum of the total wage bill (W) plus total profits (Π) [Samuelson, 1979]. This is:

$$Q_t = W_t + \Pi_t = w_t L_t + r_t K_t \, , \tag{1}$$

where w is the average real wage rate, L is total employment, r is the observed real profit rate (not the rental price of capital), and K is the stock of capital. This expression is simply an accounting identity that expresses how value added is divided between wages and total profits (the latter includes both pure profits and the imputed cost of capital), and does not require any assumption (for example, economic profits are zero, constant returns). In the words of Samuelson: "No one can stop us from labeling this last vector [residually computed profit returns to 'property' or to the nonlabor factor] as (RC_j), as J.B. Clark's model would permit—even though we have no warrant for believing that noncompetitive industries have a common profit rate R and use leets capital (C_j) in proportion to the ($P_j q_j - W_j L_j$) elements!" [Samuelson, 1979, 932].

To continue with the argument, totally differentiate the identity Equation (1) with respect to time and express it in growth rates. This yields:

(2) $$q_t = a_t \hat{w}_t + (1-a_t)\hat{r}_t + a_t \ell_t + (1-a_t)k_t = \varphi_t + a_t \ell_t + (1-a_t)k_t ,$$

where lowercase letters denote the growth rates of the corresponding variables (and with ^ for the wage and profit rates), $\varphi_t = a_t \hat{w}_t + (1-a_t)\hat{r}_t$, $a_t = (w_t L_t)/Q_t$ is the labor share, and $1 - a_t = (r_t K_t)/Q_t$ is the capital share.

Now *suppose* that in this economy the factor shares are constant (that is, $a_t = a$), and that the wage and profit rates grow at constant exponential rates, that is, $w_t = e^{(\hat{w}t)}$ and $r_t = e^{(\hat{r}t)}$, where "t" denotes time, and $\hat{w}$ and $\hat{r}$ denote the constant growth rates of the wage and profit rates, respectively.[4] This implies that the identity Equation (2), under these two assumptions, becomes:

(3) $$q_t = a\hat{w} + (1-a)\,\hat{r} + a\ell_t + (1-a)k_t = \varphi + a\ell_t + (1-a)k_t ,$$

where $\varphi = a\hat{w} + (1\text{-}a)\hat{r}$ is a constant. Now integrate Equation (3). This yields:

(4) $$Q_t = Ae^{\varphi t}(L_t)^a (K_t)^{1-a} ,$$

where A is the constant of integration.

What is Equation (4)? Given what we have done (that is, differentiate and integrate an identity), Equation (4) must be the identity Equation (1) rewritten under the two assumptions that the observed factor shares are constant and that the wage and profit rates grow at constant rates (Equation (4) is an identity *if and only if* the two assumptions about the shares are correct). Of course, the interesting point is that Equation (4) *resembles* the Cobb-Douglas production function with elasticities equal to the observed factor shares, and a neutral time shift.

This argument has several implications.[5] First, if the assumptions about the observed factor shares and the wage and profit rates are correct, and if one estimates an equation like Equation (4) unrestricted, it will yield a (suspicious) perfect fit with elasticities equal to the factor shares (and thus "constant returns"). On the other hand, if the assumptions are incorrect, estimation of Equation (4) will not yield perfect results (how good they are will depend on how far the two assumptions are from the reality). If this is the case, it must be because one or both assumptions are empirically wrong (and thus we fitted an incorrect functional form).[6] But this does not invalidate the argument. It simply means that we need other assumptions about the paths of the factor shares and wage and profit rates, thus potentially leading to other functional forms, such as the CES or the translog (that is, other "aggregate production functions" that are no more than particular cases of the income accounting identity [Felipe and

McCombie, 2001; 2003]). In other words, the identity $Q = wL + rK$ can always be transformed into the form $Q = \mathrm{F}(L, K, t)$, where "t" is a suitable function of time—not necessarily an exponential time trend. The estimated function F(•) will have all the properties of a neoclassical production function.

Second, since what has been estimated is simply an identity, or a very good approximation to it, *nothing* can be inferred. And from the econometric point of view, issues such as endogeneity problems and the possible inconsistency of the estimates [Levinsohn and Petrin, 2000], the presence of unit roots (and cointegration), or the estimation method, are irrelevant. It is an identity!

Finally, Equations (3) and (4) indicate that the putative elasticities must add up to one ("constant returns to scale"), and that they must be equal to the factor shares ("perfect competition"). No other result is possible. But is this the result of Euler's theorem? Does this imply that the economy is characterized by constant returns and competitive markets? "Nonsense," Samuelson [1979, 933] claimed. This is purely the result of the accounting identity. We will return to this issue in a later section.

This analysis also leads us to questioning the standard interpretation of the coefficient of the time trend as a proxy for the rate of technological progress. If the aggregate production function does not exist because of the aggregation problems, on what grounds is such a coefficient a measure of the rate of technical progress? What we know with certainty, because it follows from the identity Equation (4), is that the said coefficient equals $\lambda = \varphi = a\hat{w} + (1-a)\hat{r}$ (under the assumptions stated). This magnitude is simply a weighted average of the growth rates of the wage and profit rates, where the weights are the observed factor shares. This is a measure of distributional changes [Shaikh, 1980], although not necessarily in a zero-sum sense.

Alternatively, suppose that instead of fitting econometrically the aggregate production function, one carries out a growth accounting exercise. For this, one would assume that a flexible aggregate production function $Q_t = \mathrm{A}(t)\mathrm{F}(L_t, K_t)$ exists, where $\mathrm{A}(t)$ is the level of technology (in general it need not be neutral). Expressing it in growth rates and further assuming profit maximization and perfectly competitive markets it yields $q_t = \lambda_t + a_t\ell_t + (1 - a_t)k_t$, where λ_t is the growth rate of technical progress [Solow, 1957]. Note, however, that this expression is identical with Equation (2), the identity in growth rates, which was derived without making any assumption and any reference to a production function. Overall, this analysis supports Samuelson's [1979, 935-36] critical evaluation of the "residual" studies.[7]

After acknowledging the criticisms for his time series work, in particular that the regressions were fragile after dropping a number of years, Douglas moved on to cross section estimation. He thought that his results were much more robust: "It is hard to believe that these estimates can be purely accidental..." [Douglas, 1948, 40-41]. However, Samuelson [1979, 932-34] concluded that they also followed purely as a cross-sectional tautology based on the residual computation of the nonwage share. It is easy to show that a Taylor series approximation of the value added accounting identity written for a cross section, and assuming low dispersion of factor shares, yields a form that resembles a Cobb-Douglas production function [Felipe, 2001a]. In this case, the transformation of the cross-section value-added identity $Q_i = w_iL_i + r_iK_i$ yields:

(5) $$\ln Q_i \equiv B + \bar{a}\ln w_i + (1-\bar{a})\ln r_i + \bar{a}\ln L_i + (1-\bar{a})\ln K_i\,,$$

where $B = \ln\bar{Q} - \bar{a}\ln\bar{w} - (1-\bar{a})\ln\bar{r} - \bar{a}\ln\bar{L} - (1-\bar{a})\ln\bar{K}$, is a constant and a bar over the corresponding variable indicates the average of the cross section.

The Cobb-Douglas for a cross section is:

(6) $$Q_i = AL_i^{\alpha}K_i^{\beta}\,.$$

If one now estimates a logarithmic regression of output on labor and capital for a cross section of industries, regions, or countries, it is obvious that if $N = B + \bar{a}\ln w_i + (1-\bar{a})\ln r_i$ in Equation (5) is approximately constant (if w_i and r_i do not vary too much), the regression, which takes the form of Equation (6), will work econometrically. If that is the case, one will find $a = \bar{a}, b = (1-\bar{a})$. As we have argued before, since there is no reference to a production function in the derivation of Equation (5), the econometric results should not be interpreted as those stemming from any such function.

A REEVALUTAION OF THE EMPIRICAL EVIDENCE AND A PARSIMONIOUS EXPLANATION

The second step in answering the questions posed at the end of the second section is to provide empirical evidence. First, one must realize that, as shown above, the equations estimated in Table 2 can be derived from the income identity. In order to derive Equation (4) from the identity, we made two assumptions about the data. First, that the observed factor shares are constant; and second, that wage and profit rates grow at constant rates. If we had data on factor shares, both hypotheses could be tested. Since we do not, the most we can do is conjecture. Most likely, the first assumption is correct. Although factor shares were not exactly constant for the period of estimation, probably they were *sufficiently* constant for regression purposes.[8] The second assumption is the one that is, most likely, incorrect, and the one that makes the regression with the time trend turn out with such "inexplicable" results. It is not true that wage and profit rates increased at a constant rate. This implies that the exponential time trend provides a poor approximation to the evolution of φ_t and its inclusion in the regression biases the estimates of the elasticities.

The path of φ_t is simply an empirical issue. Once we approximate it, we would plug it into Equation (2) and proceed as above. We have graphed $\varphi_t = a_t\hat{w}_t + (1-a_t)\hat{r}_t$ for a series of plausible values. It displays a saw tooth shape around zero. Thus, for example, a trigonometric function with sines and cosines should provide a much better approximation than that provided by the simple linear time trend (nothing in neoclassical economics says that "technical progress" must be approximated through a linear time trend). Through trial and error we fitted the first regression in Table 3, which includes as a regressor the variable $A(t) = [\sin(T^5) + \cos(T^4) - \cos(T^2) - \sin(T^2)]$ (where T denotes time, "sin" is the sine function, and "cos" is the cosine function), with estimated coefficient $\lambda = 0.032$, statistically significant. Surely this approximation can still be improved.

TABLE 3
Cobb-Douglas Regression, II
(1899-1922 unless otherwise indicated;OLS estimates unless otherwise indicated)

1. $\ln Q_t = \lambda A(t) + \alpha \ln L_t + \beta \ln K_t$				λ		α	β
				0.032		0.726	0.274
				(3.48)		(18.83)	(7.71)
$R^2 = 0.973$; D.W. = 1.95; $\chi^2_1 = 0.02$							
2. $q_t = \gamma_1 \ell_t + \gamma_2 k_t + \gamma_3 q_{t-1} + \gamma_4 \ell_{t-1} + \gamma_5 k_{t-1} + \gamma_6 \ln VA_{t-1} + \gamma_7 \ln L_{t-1} + \gamma_8 \ln K_{t-1}$							
γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8
1.00	0.98	0.31	–0.56	–0.95	–0.78	0.59	0.19
(8.18)	(1.70)	(1.65)	(–2.19)	(–1.72)	(–3.24)	(3.24)	(2.76)
$\theta_L = 0.758$ (14.95); $\theta_K = 0.249$ (5.30); $R^2 = 0.952$; D.W. = 2.31; $\chi^2_1 = 0.56$							
3. ESTIMATION PERIOD 1899-1920: $\ln Q_t = \lambda A(t) + \alpha \ln L_t + \beta \ln K_t$				λ		α	β
				0.023		0.756	0.246
				(2.50)		(15.84)	(5.52)
$R^2 = 0.977$; D.W. = 1.76; $\chi^2_1 = 0.43$							
4. IN PER CAPITA TERMS: $\ln(Q_t/L_t) = \lambda A(t) + (\alpha + \beta - 1)\ln L_t + \beta \ln(K_t/L_t)$				λ		$\alpha + \beta - 1$	β
				0.029		0.001	0.259
				(2.39)		(0.43)	(6.64)
$R^2 = 0.768$; D.W. = 1.95							
5. NON-LINEAR LEAST SQUARES: $Q_t = e^{\lambda A(t)} L_t^{\alpha} K_t^{\beta} + u_t$				λ		α	β
				0.033		0.722	0.277
				(3.65)		(16.12)	(6.80)
$R^2 = 0.964$; D.W. = 1.90; $\chi^2_1 = 0.00012$							

Notes: Chi-square test (χ^2_1): H_0:$\alpha + \beta = 1$ (critical value 5 percent significance level: 3.84). t-statistics in parentheses. Initial values for nonlinear least squares: $\lambda = 0.03$; $\alpha = 0.75$; $\beta = 0.25$.

Why does A(t) work? Assume in Equation (2) above that the factor shares α_t and $(1 - \alpha_t)$ are constant and integrate it. This leads to $Q_t = (w_t)^{\alpha} (r_t)^{1-\alpha} (L_t)^{\alpha} (K_t)^{1-\alpha}$. If indeed factor shares were exactly constant, this expression would be the identity, and so all A(t) in the first regression in Table 3 does is approximate the term $(w_t)^{\alpha} (r_t)^{1-\alpha}$. We can therefore compute the value of $(w_t)^{\alpha} (r_t)^{1-\alpha}$ through the ratio $Q_t / (L_t)^{\alpha} (K_t)^{1-\alpha}$. The graph of this ratio is shown in Figure 2, and the approximation through $A(t) = [\sin(T^5) + \cos(T^4) - \cos(T^2) - \sin(T^2)]$ is given in Figure 3. Although the approximation is not perfect (the correlation between A(t) and $Q_t / (L_t)^{\alpha} (K_t)^{1-\alpha}$ is 0.588), it is certainly much better than that provided by the exponential time trend and, as argued above, it suggests that finding the exact path is simply a matter of trial and error and a dose of patience in front of a computer.[9]

FIGURE 2
$Q/[(L^{0.75})*(K^{0.25})]$

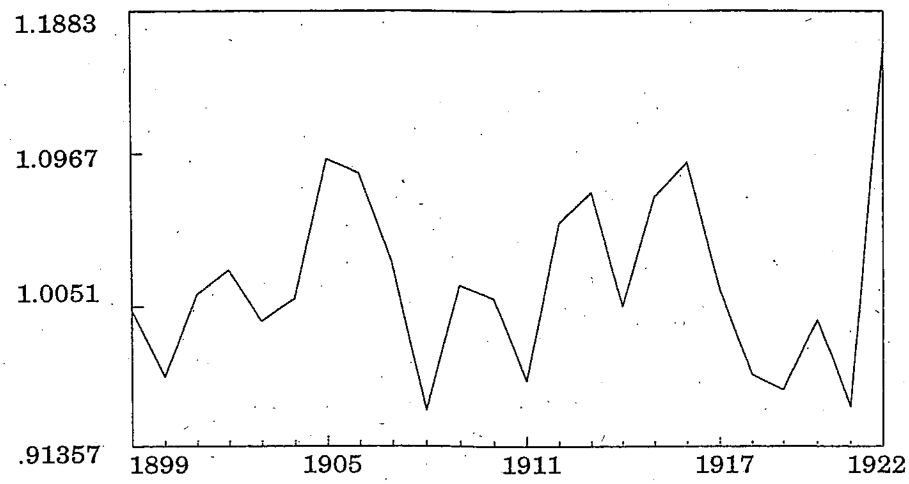
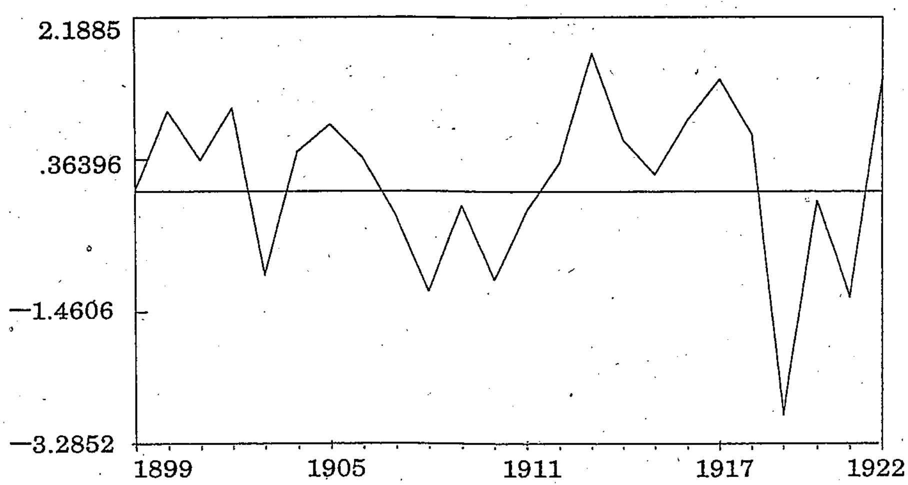


FIGURE 3
$A(t) = \sin(T^5) + \cos(T^4) - \cos(T^2) - \sin(T^2)$



Summing up: what was the problem with the regression with the linear trend? While it appears that the factor shares were sufficiently constant for the Cobb-Douglas form to work as a way to approximate an accounting identity, the linear trend was a bad choice to approximate the weighted average of the wage and profit rates.

Regarding the equation in growth rates, the second equation in Table 3 shows a good approximation to Equation (2) (note the increase in fit with respect to the regression in growth rates in Table 2). Notice that this is a dynamic regression. The interesting aspect of this regression is that it can be easily derived as a dynamic parameterization of a Cobb-Douglas production function in levels with two lags [Bårdsen, 1989]. The "long-run" output elasticities of labor and capital are given by $\theta_L = -(\gamma_7/\gamma_6)$ and $\theta_K = -(\gamma_8/\gamma_6)$, respectively.[10] Their values (with the t-values in parentheses) are provided in the following row, together with the summary statistics. Once again, they equal the factor shares. And notice that the negative sign on the stock of capital has disappeared. Pesaran, Shin, and Smith [1999] have proposed a framework to test whether there exists a long-run relationship among a number of variables within the current framework, irrespective of whether the variables are integrated of order zero, I(0), or of order one, I(1). The test is an F-statistic for the significance of the lagged levels of the variables in the autoregressive distributed lag, that is, H_0: $\gamma_6 = \gamma_7 = \gamma_8$. Pesaran, Shin, and Smith [1999] have tabulated the appropriate critical values for different numbers of regressors, and have provided a band of critical values assuming that the variables are I(0) or I(1). The result of the test yields F(3, 14) = 5.16. In our case, the corresponding band of critical values for a significance level of 0.05 is 3.79 to 4.85 [Pesaran, Shin, and Smith, 1999, Table C1.iii]. Since our calculated F-test exceeds the upper bound of the band, we reject the null hypothesis of no long-run relationship among output, labor, and capital. These results, from the strict econometric point of view, imply that a long-run relationship exists among the three variables. However, in the context of an approximation to an accounting identity, this result does not have any deep economic interpretation: it is the accounting identity in disguise.

The importance of the above results can be further appreciated by recalling the poor estimates obtained in the fifth regression in Table 2, when the original Cobb-Douglas equation was estimated for 1899-1920. Estimating the third regression in Table 3 with the variable A(*t*) for 1899-1920 does not yield fragile results, however. This result is corroborated by the forward and backward recursive estimation of this equation, shown in Tables 4 and 5, respectively. This is, of course, precisely what we should expect, as the specification of the putative production function is now a close approximation to the underlying identity.

We have also estimated the Cobb-Douglas form in per capita terms including the trigonometric variable. The fourth regression in Table 3 shows the significant improvement after its inclusion in the regression (compare it with the second regression in Table 2). The estimate of labor provides a direct test for the null hypothesis of constant returns, which cannot be rejected.

Finally, we estimated the regression using nonlinear least squares, as in Pesaran and Pesaran [1997, 251-53]. These results are shown at the bottom of Table 3. The results are very similar to those using ordinary least squares, indicating that the estimation method is not an issue.

A TEST OF CONSTANT RETURNS TO SCALE AND PERFECTLY COMPETITIVE MARKETS?

Douglas was so convinced of the importance of his analysis that towards the end of his life he concluded that a "considerable body of independent work tends to corroborate the original Cobb-Douglas formula, but, more important, the approximate coincidence of the estimated coefficients with the actual shares received also strengthens the competitive theory of distribution and disproves the Marxian" [Douglas 1976, 914]. In this vein, Solow [1974, 121] pointed out that: "When someone claims that aggregate production functions work, he means (a) that they give a good fit to input-output data without the intervention of factor shares and (b) that the function so fitted has partial derivatives that closely mimic observed factor shares."[11] It is thus implicit that it is possible to test whether the partial derivatives, that is, the first-order conditions, closely approximate the factor shares.

TABLE 4
Forward Recursive Estimation of the Equation
$\ln Q_t = \lambda[\sin(T^5) + \cos(T^4) - \cos(T^2) - \sin(T^2)] + \alpha \ln L_t + \beta \ln K_t$

Period	λ	α	β	H_0: $\alpha + \beta = 1$	R^2; D.W.
1899-1903	0.0025	0.525	0.472	0.09	0.938;
	(0.10)	(0.57)	(0.51)		2.54
1899-1904	0.0005	0.665	0.333	0.12	0.946;
	(0.03)	(3.09)	(1.57)		2.40
1899-1905	0.008	0.405	0.591	0.52	0.938;
	(0.40)	(1.83)	(2.71)		2.51
1899-1906	0.007	0.340	0.656	0.81	0.955;
	(0.40)	(2.00)	(3.94)		2.48
1899-1907	0.014	0.433	0.563	0.63	0.962;
	(0.79)	(3.05)	(4.06)		2.47
1899-1908	0.033	0.674	0.324	0.007	0.919;
	(1.45)	(4.59)	(2.28)		1.66
1899-1909	0.033	0.677	0.322	0.007	0.934;
	(1.56)	(5.47)	(2.69)		1.81
1899-1910	0.031	0.668	0.331	0.006	0.943;
	(1.59)	(5.77)	(2.97)		1.80
1899-1911	0.026	0.736	0.265	0.051	0.937;
	(1.29)	(6.70)	(2.51)		1.59
1899-1912	0.030	0.705	0.295	0.004	0.948;
	(1.67)	(7.46)	(3.25)		1.98
1899-1913	0.029	0.707	0.293	0.007	0.958;
	(2.26)	(9.33)	(4.06)		2.00
1899-1914	0.027	0.733	0.267	0.09	0.959;
	(2.15)	(10.79)	(4.14)		1.88
1899-1915	0.028	0.705	0.294	0.007	0.962;
	(2.23)	(11.36)	(5.00)		2.05
1899-1916	0.031	0.689	0.310	0.002	0.970;
	(2.53)	(11.72)	(5.58)		2.00
1899-1917	0.025	0.713	0.287	0.038	0.973;
	(2.16)	(12.43)	(5.31)		2.01
1899-1918	0.023	0.749	0.252	0.27	0.971;
	(1.85)	(12.92)	(4.62)		1.62
1899-1919	0.023	0.749	0.253	0.29	0.974;
	(2.45)	(14.12)	(5.09)		1.77
1899-1920	0.023	0.756	0.246	0.43	0.977;
	(2.50)	(15.84)	(5.52)		1.26
1899-1921	0.024	0.775	0.228	0.98	0.976;
	(2.65)	(19.15)	(6.08)		1.71
1899-1922	0.032	0.726	0.274	0.02	0.973;
	(3.48)	(18.83)	(7.71)		1.95

Chi-square test (χ^2_1): H_0: $\alpha + \beta = 1$ (critical value 5 percent significance level: 3.84). t-statistics in parentheses.

TABLE 5
Backward Recursive Estimation of the Equation
$\ln Q_t = \lambda[\sin(T^5) + \cos(T^4) - \cos(T^2) - \sin(T^2)] + \alpha \ln L_t + \beta \ln Kt$

Period	λ	α	β	H_0: $\alpha + \beta = 1$	R^2; D.W.
1918-1922	0.035	0.591	0.389	0.37	0.738;
	(1.42)	(2.60)	(1.98)		2.50
1917-1922	0.034	0.582	0.396	0.99	0.746;
	(1.88)	(3.69)	(2.88)		2.64
1916-1922	0.037	0.659	0.331	0.23	0.639;
	(1.99)	(4.45)	(2.55)		2.21
1915-1922	0.036	0.690	0.300	0.05	0.693;
	(2.05)	(5.02)	(2.52)		1.85
1914-1922	0.036	0.677	0.317	0.17	0.816;
	(2.19)	(5.52)	(2.93)		2.07
1913-1922	0.037	0.681	0.313	0.17	0.835;
	(2.50)	(6.20)	(3.22)		2.12
1912-1922	0.036	0.701	0.290	0.03	0.849;
	(2.53)	(7.18)	(3.31)		1.99
1911-1922	0.038	0.683	0.311	0.26	0.890;
	(2.84)	(7.63)	(3.90)		2.17
1910-1922	0.033	0.712	0.285	0.02	0.900;
	(2.76)	(8.75)	(3.92)		2.17
1909-1922	0.033	0.719	0.279	0.00	0.915;
	(2.87)	(9.70)	(4.21)		2.16
1908-1922	0.035	0.710	0.287	0.05	0.941;
	(3.30)	(10.05)	(4.52)		2.21
1907-1922	0.034	0.729	0.271	0.00	0.943;
	(3.31)	(11.33)	(4.66)		2.22
1906-1922	0.035	0.755	0.248	0.33	0.942;
	(3.33)	(12.41)	(4.50)		2.01
1905-1922	0.036	0.771	0.233	0.91	0.944;
	(3.50)	(13.55)	(4.52)		1.91
1904-1922	0.036	0.760	0.243	0.63	0.953;
	(3.54)	(12.24)	(5.01)		2.00
1903-1922	0.035	0.765	0.239	1.04	0.959;
	(3.73)	(16.15)	(5.54)		2.03
1902-1922	0.034	0.754	0.49	0.71	0.963;
	(3.73)	(12.22)	(6.21)		2.03
1901-1922	0.034	0.748	0.254	0.57	0.968;
	(3.81)	(18.42)	(6.82)		2.02
1900-1922	0.032	0.726	0.273	0.02	0.968;
	(3.39)	(17.53)	(7.17)		1.82
1899-1922	0.032	0.726	0.274	0.02	0.973;
	(3.48)	(18.83)	(7.71)		1.95

Chi-square test (χ_1^2): H_0: $\alpha + \beta = 1$ (critical value 5 percent significance level: 3.84). t-statistics in parentheses.

We pose the following question: Is there any way that estimation of the aggregate production function or the marginal productivity conditions can indicate the existence of imperfect markets and returns to scale different from constant? The answer is clearly no. At the expense of laboring the obvious, if one runs the putative production function regression of output (q_t) on the growth rates of labor (ℓ_t) and capital (k_t), and the *correct approximation* to φ_t, Equation (2) indicates that the estimated coefficients of ℓ_t and k_t must be the factor shares (same argument with the equation in levels). The only way not to obtain this result is if the approximation to φ_t is incorrect; for example, if it is a trigonometric function and one chooses a constant (as we saw above). In the case at hand, the Cobb-Douglas form works because factor shares must be sufficiently constant. All the "correct" production-function regressions that we have estimated indicate constant returns to scale and perfectly competitive markets, including in countries like Singapore [Felipe, 2000; Felipe and McCombie, 2003] or China [Felipe and McCombie, 2002a].

What if, instead of estimating the production function, we estimate the first-order conditions? This analysis also implies that these conditions cannot be rejected. Under the assumptions of profit maximization and competitive markets, the production function, together with the assumption that firms are profit maximizers, gives rise to the marginal theory of factor pricing. This analysis, which is strictly microeconomic [Fisher, 1971b], has been equally applied to the macro level in the form of a distribution theory. The first-order condition for labor states that the wage rate equals the marginal product of labor: $w = \partial Q/\partial L$ (recall that at the aggregate level the measure of output is value added). And the labor share equals the elasticity of labor: $wL/Q = (L/Q)(\partial Q/\partial L)$.[12]

Consider again the identity $Q = wL + rK$. It follows that $w \equiv \partial Q/\partial L$. How can this be posed as a testable proposition? For the Cobb-Douglas production function, the first-order condition for labor is $w_t = a(Q_t/L_t)$. Although we do not have the wage rate data for the Cobb-Douglas [1928] data set, we know that this "hypothesis" cannot be rejected. The reason is that the last relation cannot be distinguished statistically from the definition of the labor share $a_t \equiv (w_t L_t)/Q_t$ if $a_t = a$. Since we have argued that for this data set factor shares must be (sufficiently) constant, it is obvious that estimation of the regression $w_t = \gamma_1(Q_t/L_t)$ must yield $\gamma_1 = a$. But this does not provide evidence in favor of the competitive theory of distribution. It is a tautology! We have checked this using data for the U.S. manufacturing sector for 1960-94 (OECD database). The regression of the wage rate (w_t) on labor productivity (Q_t/L_t) yields a coefficient of 0.688 (with a t-statistic of 168.00), statistically not different from the average labor share ($\bar{a} = 0.692$).[13]

If the exercise of estimating an aggregate production function (or the first-order conditions) is correctly performed, one should *always* be led to believe that the evidence indicates that markets are perfectly competitive, and that the production function is homogeneous of degree one. Consequently, constant returns to scale and perfect competition are nonrefutable hypotheses.[16]

CONCLUSIONS

This paper has taken up Samuelson's [1979] invitation to verify empirically his claim that all the regression of the Cobb-Douglas [1928] production function does is to reproduce the income accounting identity according to which value added equals the sum of the wage bill plus total profits. We conclude that Samuelson was right, and believe that this argument has very serious implications for today's work in macroeconomics.

We have shown that since the data on output and inputs used at the aggregate level are linked through the accounting identity that relates value added and factor payments, aggregate production functions approximate this income accounting identity. An algebraic transformation of the identity, under the appropriate assumptions about the data, yields a form that resembles a production function. This implies that if the correct form of the identity, written as a production function, were fitted, one should always conclude that the aggregate production function exhibits constant returns to scale, and that factor markets are competitive. Surely this would be a suspicious result. The important aspect of this argument is that it can parsimoniously explain why, despite the fact that aggregate production functions do not have a sound theoretical basis, they appear to yield meaningful results at times. Likewise, the poor results that quite often appear (for example, when a linear time trend is added) are no more than the result of a poor approximation to the income accounting identity.

The conclusion is that neither the existence of the aggregate production function, nor the standard neoclassical hypotheses of constant returns to scale or competitive markets, can be tested empirically since they cannot be refuted.

NOTES

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1. This is the title of Cobb and Douglas's original article in 1928.
2. For a recent review of the Cambridge debates see Cohen and Harcourt [2003].
3. Some of the same points made in this section were previously made by McCombie [1998].
4. The assumption of constant factor shares need not imply a Cobb-Douglas production function. Fisher [1971a], in a simulation study, showed that a very close statistical fit could be obtained by estimating an aggregate Cobb-Douglas production function, even though the data were such that the conditions for successful aggregation of the underlying micro Cobb-Douglas production functions were deliberately violated. He showed that, in these circumstances, the constancy of the factor shares gave rise to the success of the aggregate Cobb-Douglas production function, rather than *vice versa*. He concluded: "the view that constancy of labor's share is due to the presence of and aggregate Cobb-Douglas production function is mistaken. Causation runs the other way and the apparent success of aggregate Cobb-Douglas production functions is due to the relative constancy of labor's share" [Fisher 1971a, 306]. And Samuelson wrote: "A follower of Douglas might wish to derive the comfort from the fact that, in many different times, a Bowley will report pretty much the same relative wage share for a particular country like the United States or the United Kingdom. But why cannot such a fact, or alleged fact, stand on its own bottom, gaining and losing nothing from being coupled with an aggregate neoclassical production function?" [Samuelson 1979, 931].
5. The same derivation applies if the measure of output is gross output. In this case we have to write the identity for gross output, and proceed with a similar derivation. It leads to a production function with gross output on the left-hand side, and labor, capital, and intermediate materials on the right-hand side, with the shares of labor, capital, and intermediate materials in gross output as elasticities.
6. Naturally, in this case it will make a difference whether the regression is estimated in levels or in growth rates, as well as the estimation method. These are, nevertheless, secondary problems and do not affect the generality of the argument.

7. It must be emphasized that these results stem from the fact that we are dealing with aggregates, which can only be expressed in *value* terms (certainly quantity indices, as defined above, are not physical volumes, but the ratio of two values). The argument above does not apply if output and inputs were measured in physical units. See Felipe and McCombie [2005] in this issue.
8. This assumption could be tested easily by fitting the left-hand side of Equation (2) unrestricted, that is, $q_t = \gamma_1 \hat{w}_t + \gamma_2 \hat{r}_t + \gamma_3 \ell_t + \gamma_4 k_t$ and testing whether the coefficients equal the average factor shares (that is, H_0: $\gamma_1 = a$; $\gamma_2 = 1 - a$; $\gamma_3 = a$; $\gamma_4 = 1 - a$).
9. Still, at this point one may argue that all we are doing is inserting back into the equation the "Solow residual" and, therefore, we should expect a perfect fit. This argument faces two objections. First, what we are inserting is not the Solow residual itself, but a function of sines and cosines that tracks such residual better than the linear time trend that is usually introduced. Second, the exercise shows that once this function is found, we recover the identity and, by implication, the elasticities equal the factor shares (always!). See Shaikh [1980, 86].
10. This dynamic Cobb-Douglas is $Q_t = A(Q_{t-1})^{\delta_1}(Q_{t-2})^{\delta_2}(L_t)^{\alpha_1}(L_{t-1})^{\alpha_2}(L_{t-2})^{\alpha_3}(K_t)^{\beta_1}(K_{t-1})^{\beta_2}(K_{t-2})^{\beta_3}$, where the long-run output elasticities are given by $\theta_L = -\gamma_7/\gamma_6 = -(\alpha_1 + \alpha_2 + \alpha_3)/(\delta_1 + \delta_2 - 1)$ and $\theta_K = -\gamma_8/\gamma_6 = -(\beta_1 + \beta_2 + \beta_3)/(\delta_1 + \delta_2 - 1)$. And the dynamic regression rewritten with the error correction term is: $q_t = c + \gamma_1 \ell_t + \gamma_2 k_t + \gamma_3 q_{t-1} + \gamma_4 \ell_{t-1} + \gamma_5 k_{t-1} + \gamma_6 [\ln Q_{t-1} + \theta_L \ln L_{t-1} + \theta_K \ln K_{t-1}]$. The long-run solution is $Q_t = \Psi(L_t)^{(\alpha_1+\alpha_2+\alpha_3)/(1-\delta_1-\delta_2)}(K_t)^{(\beta_1+\beta_2+\beta_3)/(1-\delta_1-\delta_2)}$, where Ψ is a constant. Compare this expression now to the identity Equation (2) under the assumption of constant factor shares. This is (integrating): $Q_t = (w_t)^a (r_t)^{1-a} (L_t)^a (K_t)^{1-a}$. If the term $(w_t)^a (r_t)^{1-a}$ happens to be a constant A, this becomes $Q_t = A(L_t)^a (K_t)^{1-a}$. If, as discussed above, this constant works empirically, no wonder one will find $(\alpha_1 + \alpha_2 + \alpha_3)/(1 - \delta_1 - \delta_2) \cong a$, and $(\beta_1 + \beta_2 + \beta_3)/(1 - \delta_1 - \delta_2) \cong (1 - a)$.
11. This paper is a severe criticism and dismissal of Shaikh [1974]. We ask the reader to see Shaikh [1980] for a full reply to Solow.
12. On this see also Felipe [2001b] and Felipe and McCombie [2002b].
13. It is far from surprising that recent time series work does find the existence of a long-run relationship between the wage rate and labor productivity based on the regression $\ln w_t = c + \delta \ln(VA_t/L_t)$ with $\delta = 1$ [Darby and Wren-Lewis, 1993]. That is exactly the coefficient in the labor share identity, and thus it means nothing.
14. Dhrymes [1965] proposed to estimate the degree of homogeneity parameter from the equation $w = AQ^{\beta}L^{\gamma}$ (estimated in logarithms), where w is the wage rate, Q is output, and L is labor (the equation is derived from a CES production function). The degree of homogeneity h is calculated from the estimates as $h = (1 + \gamma)/(1 - \beta)$. This equation suffers from exactly the same problem discussed above, however, namely, that it can be derived from an identity. To see this, note that the definition of the labor share is $a_t = (w_t L_t)/Q_t$, where, as before, "a" is the labor share. Assume that in this economy the labor share is constant. This expression can then be rewritten as $w_t = aQ_t L_t^{-1}$, which is identical to Dhrymes' [1965] regression. What this result indicates is that the regression of the log wage rate on log output and log labor must yield coefficients $\beta = 1$ and $\gamma = -1$, unless the labor share has a large variation, in which case the regression results will be poor. But with these theoretical values for β and γ, the degree of homogeneity implied by this regression is $h = (1 + \gamma)/(1 - \beta) = 0/0$, indeterminate.

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